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INPUT-OUTPUT TABLES AND ANALYSIS

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PREFACE

This publication brings up to date and amplifies the technical manual, Problems of Input-Output Tables and Analysis, Studies in Methods, Series F, No. 14, that was issued in 1966.[1]

As the Statistical Commission recommended, it makes use of the treatment and terminology of input-output tables in the present United Nations System of National Accounts (SNA);[2] this system had not been fully worked out or accepted by the Commission at the time the 1966 publication was prepared and issued. Basic input-out tables are dealt with as an integral part of national accounting, in particular, as the way in which the production accounts of the SNA are disaggregated. The present publication is thus designed to be a technical manual on input-output tables in the context of the SNA as well as in general.

The publication also reflects other developments in the work on input-output data since the 1966 document was prepared. It deals with such topics as rectangular tables, techniques for updating input-output data, dynamic input-output and the uses of input-output data in the analysis of investment and capital requirements, in measuring productivity and in planning economic growth. It also clarifies and amplifies the discussion in the 1966 publication of such questions as the treatment of secondary production, the valuation of transactions and the compilation of data on imports.

The publication deals with the framework and concepts of input-output tables and analysis, the methodology of, and sources of basic data for, compiling the tables, and the uses to which these data may be put. Chapters I and II concern the SNA framework for, and the concepts, elements, interrelationships and tabulations of, input-output data and the assumptions underlying input-output analysis. Chapters III and IV deal with the methodology of, and sources of basic data for, input-output tables. Chapter V discusses the various ways in which input-output data may be analysed and used. A comparative tabular analysis of national practices in the compilation of input-output tables is also included in this publication; the tables are set out in annex II.

Except for annex II, this publication was prepared by A.G. Armstrong, who served as consultant to the Secretariat.

1/ United Nations publication, Sales No. 66.XVII.8.

2/ A System of National Accounts, Studies in Methods, Series F, No. 2, Rev. 3 (United Nations publication, Sales No. E.69.XVII.3).

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I. INPUT-OUTPUT AND NATIONAL ACCOUNTS

A. Introduction

1.1. There are two key features in the System of National Accounts (SNA)[1/] published in 1968, both of which are very relevant to the estimation and application of input-output tables. In the new system all the accounts are presented in a matrix layout and all transactions are recorded as between two sectors or categories of an account. Secondly, the new system incorporates the recommendations of earlier reports which were not considered practical before this date and makes an input-output table an integral and important part of the statistical framework of national accounting. Thus, not only does the new system extend the basic principles of the input-output table to all other transactions in the economy but it also emphasises the key role which input-output can play in national accounting and the important link between the two.

1.2. An input-output table obviously itself contains much useful information (the application of which is discussed at length in this volume) but it also completes the statistical framework and provides the link between many other accounts. The omission of an input-output table from a statistical framework means that certain transactions are not recorded and thus it would not be possible to balance all accounts. The production accounts are particularly affected in this way and as the statistical information contained in such accounts is of such importance to the planning and management of any economy it is vital to have accurate data. This can only be obtained if the accounts are completed and balanced by the inclusion of an input-output table. This important role was well described by the French statistician, Delange: "The input-output table has played and will play an important part in the improvement of statistical data, because, since it is a means for achieving consistency, it clearly shows those statistical shortcomings which should be eliminated first."

1/ A System of National Accounts, Studies in Methods, Series F, No. 2, Rev. 3 (United Nations publication, Sales No. E.69.XVII.3).

1.3. Most national statistical offices have their own examples of the problems of achieving balance and consistency between various items in national accounting. Typical of this at the aggregate level is the problem of ensuring that the output,income and expenditure methods of measuring national income yield the same result. When dealing with disaggregated accounts as in the S.N.A. the problems become more numerous and are of the type of ensuring that the recorded demands for the products of each sector are equal to the recorded supplies. Such problems become more difficult when one is dealing with movements in such statistics over a period of years and not only achieving balance in one particular year, for it is then necessary that all price and volume indicators are internally consistent. This balance and consistency will not be achieved if the input-output table which accounts for approximately half of total demand for industrial output is excluded.

1.4. Thus, it is important that a national accounting framework should include an input-output table as an integral part. It is also important that the estimation of an input-output table is not carried out independently of the national accounts and, indeed, most national input-output tables balance with relevant national income data. The recognition of this two-way link between input-output and national accounts is vital for anyone involved in the collection of statistical data or the analysis of economic relationships. An input-output table should not be regarded as something in itself but as part of the whole system. It is true that a table contains much interesting information about the inter-dependence of various industrial sectors but its main role comes when this information is used to trace relations through the system and e.g. provide a link from final demand to industrial outputs and primary inputs or between the price of imported materials or wage rates and the price of finished goods.

B. An aggregated social accounting matrix

1.5. Before discussing the basis of input-output analysis in the following chapter it will be useful to look briefly at the accounting framework. An aggregated social accounting matrix relating to the United Kingdom for 1963 is shown in Table 1.1. The accounts shown in this table are similar to those in Table 1.6 of the S.N.A. but the accounts relating to assets have been omitted and the accumulation account is shown here in less detail. The arrangement and methodology of this accounting matrix is fully described in the S.N.A. and will not be discussed at length here. It will however, be useful to outline briefly the principles of this matrix presentation of accounts since they apply also to the input-output table and since they enable one to see the input-output table in relation to the rest of the accounts.

Table 1.1. Aggregated social accounting matrix for United Kingdom, 1963
(£ hundred million)

			Production		Consumption		Accumulation		World	Totals
			1	2	3	4	5	6	7	
Production	Commodities	1		242	220		51		56	569
	Industries	2	483							483
Consumption	Consumer goods	3				254			2	256
	Income and outlay	4	35	237	31			-23	4	284
Accumulation	Capital formation	5						51		51
	Capital finance	6				29				29
Rest of the world		7	51	4	5	1		1		
Totals			569	483	256	284	51	29	62	

1.6. The outgoings from each account - the expenditure of that sector - are shown in the columns and the receipts by each account are shown in the rows. Money is thus flowing from the account denoted in the column to the account of the row and the location of each entry within the matrix denotes the two sectors involved in each transaction. There are flows of goods, services or financial assets in the opposite direction to each of the money flows. For instance, in the first row of Table 1.1 it can be seen that commodities (goods and services) are purchased by industries and consumers and for capital formation and export. An important feature of this system is that all transactions by each sector are recorded and total outgoings equal total receipts for each of the sectors, i.e. all the accounts balance.

1.7. These principles will apply if the accounts are shown in the aggregated form of Table 1.1 or if some of the accounts are disaggregated as with the production account in input-output work. National accounts can be shown in this aggregated form or the various accounts can each be disaggregated as in Table 2.1 in the S.N.A. Alternatively, part of the system can be highly disaggregated and the remainder of the accounts can be aggregated even more than in Table 1.1. This is the case with input-output accounting where attention is concentrated on the production accounts, and in particular on the intra-production accounts, and where one is not greatly concerned with the transactions which do not involve the production accounts.

C. The accounting framework for input-output

1.8. A social accounting matrix arranged in a manner most appropriate for input-output purposes is shown in Table 1.2. Here, the commodity and industry accounts are disaggregated and other accounts are treated in various ways. The capital finance account, for instance, can be omitted and other accounts need only be disaggregated where they meet the production accounts. The size of the cells in Table 1.2 and the marking on the cells indicates where there is disaggregation and which entries will appear as matrices, as rows or columns (i.e.vectors) or as single numbers. The amount of disaggregation of cells shown here is the minimum requirement. Greater disaggregation may be required for some purposes as described in Chapter III.

1.9. It will be desirable to explain the various entries in this matrix of accounts but before doing so the distinction between industries and commodities should be made clear. The disaggregated classification of both is the same, commodities being defined as the characteristic or principal products of the corresponding industries. Producing units are classified to a particular industry dependent on which commodity they produce or, if they produce more than one commodity, which commodity accounts for the greatest part of their output. Many producing units, in fact, produce only a single commodity, but a number will be found which also produce commodities which are the characteristic product of other

Table 1.2. Simplified accounting framework for input-output

		Production			Consumption expenditure		Accumulation		Rest of the world
							Capital formation		
		Commodities	Commodity taxes	Industries	Personal	Government	Stocks	Fixed capital	
Production	Commodities			1	2	3	4	5	6
	Commodity taxes			7	8	9	10	11	12
	Industries	13							
Consumption (income and outlay)	Indirect taxes	14	15						
	Factor incomes			16	17	18			19
Accumulation									
Rest of the world		20		21	22	23			

industries. It is thus impossible to define the elements in the two classifications in such a way that there is a one-to-one correspondence between them. This is important since what is observed about sales structures usually relates to commodities and what is observed about cost structures usually relates to industries. This has consequences when analysing the effects on the productive system of variations in the demand for different commodities since, in general, one cannot observe the cost structures for individual commodities. These problems are further discussed in Chapter III, Section B.

1.10. The initial link between the two accounts in our framework in Table 1.2 is matrix 13 which is known as the make matrix. This records the amounts of each commodity produced in each industry. Typically, the great majority of production appears in the main diagonal; this is the production by each industry of its characteristic products. The off-diagonal entries are very much smaller and relate to secondary production, i.e. the production by industries of commodities which are the characteristic product of other industries.

1.11. In describing the entries in Table 1.2 let us look first at the commodity account. Matrices 1 through 6 record the purchases of commodities by the various sectors of demand. Matrix 1, which shows the purchases of commodities by industries, is the most important for our purposes and is known as the absorption matrix. Vectors 2 through 5 record the purchases of various commodities by each of the four sectors of home final demand whilst vector 6 records commodity details of exports. Thus all demands for commodities are recorded and if we look at the commodity account column we see the sources of supply. Commodities are either produced by domestic industries in matrix 13 or imported in vector 20. Some imports may be subject to protective duties (i.e. duties levied in order to raise the cost of the import in relation to the corresponding domestic cost) and these are recorded in vector 14. It is important (as explained in Chapter III, Section F) to separate any taxes levied on the sales of commodities from the basic values of the commodities and such taxes (and subsidies) are recorded in entries 7 through 12, debited to the accounts of the various purchasers; these are transferred to the income and outlay account in cell 15.

1.12. The other account of importance in the present context is the industry account. Industries purchase commodities in matrix 1 and pay the relevant taxes in vector 7. Payments to factors are recorded in vector 16. Industries may also occur expenditure abroad in the form of spending by travelling businessmen and disbursements by shipping fleets and airlines. It is convenient to treat this as a direct import in vector 21 rather than attempt to estimate a commodity breakdown and enter in vector 20, particularly as such expenditure is, by definition, inevitably occurred abroad and could not be met from domestic production.

1.13. The entries in Table 1.2 described above are those which are required to ensure that the production accounts and the input-output table are in complete balance. It is useful also to include the other entries. Cells 17 and 18 record wage incomes paid respectively by households (e.g. to domestic servants) and by governments to their employees and cell 19 is income received from abroad; cells 16 through 19 thus give a complete record of national income. Finally, cells 22 and 23 record expenditure abroad and together with cells 20 and 21 complete the import account.

1.14. This accounting framework follows the principles of the S.N.A. but differs slightly from the S.N.A. Table 2.1 in its treatment of some entries. Incomes paid, i.e. productive activity, by households and government, are treated in a simpler manner; the treatment of taxes is also different and here is less complex. Some of the entries in the income and outlay and the accumulation accounts are omitted but all the details in the production accounts are recorded and these accounts will balance in both aggregated and disaggregated forms. A complete set of national accounts should, of course, follow the S.N.A. table but when one is working with input-output it may be found to be more convenient and quite adequate to extract the cells shown here from the main tables and to treat them and arrange them as suggested above.

1.15. The national accounts for the U.K. shown in Table 1.1 have been disaggregated and arranged for input-output purposes following the pattern of Table 1.2 and are shown in Table 1.3. Six industries and commodities are distinguished and the totals of each matrix and vector are shown in

Table 1.3. Simplified input-output table and related accounts, United Kingdom 1963 (£ hundred million)

		Commodities							Commod. Tax	Industries							Consumpt.		Cap. Form.		World	Totals
		1	2	3	4	5	6	Total		1	2	3	4	5	6	Total	Persons	Govt.	Stocks	Fixed		
Commodities	1. Agriculture, fishing and food									21	1		2		0	24	41	1	1		3	70
	2. Chemicals									3	7	4	4	2	3	24	4	2	-0	0	6	36
	3. Metals and machinery									2	1	46	2	6	5	62	8	9	1	20	23	123
	4. Other manufactures									2	1	4	23	8	7	45	21	2	0	0	7	75
	5. Mining, construction and utilities									1	7	4	2	12	4	29	12	4	-0	25	1	71
	6. Transport & services									6	4	12	7	4	19	52	85	4	0	3	16	160
	Total									35	21	70	40	32	38	236	171	22	2	48	56	535
Commodity taxes										-2	0	1	1	1	5	6	26	1	0	1	0	34
Industries	1. Agriculture, fishing and food	51	0			0	1	53														53
	2. Chemicals	1	30	0	0	1	0	31														31
	3. Metals and machinery		0	114	0	0	0	116														116
	4. Other manufactures		0	0	65	0	0	66														66
	5. Mining, construction and utilities		0	0	0	65	1	66														66
	6. Transport & services						151	151														151
	Total	51	31	114	66	66	155	483														483
Indirect taxes		0	0	1	0	0	0	1	34													
Factor incomes										20	10	45	25	33	104	237	4	27		-23	4	
Rest of the world		19	5	8	9	5	5	51		0	0	0	0	0	4	4	3	2				
Totals		70	36	123	75	71	160	535	34	53	31	116	66	66	151	483						

Notes: (1) This set of accounts with domestic and import flows shown separately for all relevant transactions appear in Table 3.3.
(2) These tables are based on the 70 sector tables in "Input-Output Tables for the U.K." prepared by the U.K.

order to facilitate comparison with the entries in Table 1.1. If the entries in the commodity account are combined with the commodity tax account, the values in the commodity account of Table 1.1, where taxes were not shown separately, will be found. Consumption and capital formation are each divided into two components in Table 1.3, unlike Table 1.1. It can be seen here that, as in Table 1.1, all the production accounts balance and further that here the row and column totals of the disaggregated production accounts are also in balance.

1.16. It will be helpful to examine the entries for one of the sectors in the production accounts, chemicals. Looking first at the commodity account, we see that the chemical industry produced 30 chemicals and that small amounts of chemicals were also produced in four other industries. There were imports of chemicals amounting to 5 on which a small amount of import duty was levied. Reading along the row for chemicals commodity we see that 3 were purchased by the agriculture and food industry, 4 by metals and machinery and that the total intermediate demand was 24. The next five entries show the amounts purchased by domestic consumers and capital formation and, finally, exported. The row total, i.e. total demand, is 36 and equals the column total, i.e. total supply.

1.17. If we look at the rows of the industry accounts we see that the chemical industry produced 30 chemicals commodity, 1 each of food and construction commodities and small quantities of the other three commodities. Its total output is 31 and looking at the industry account columns we see its input structure. The two largest inputs are 7 chemicals (for further processing) and 7 mining products etc. The total commodity input is 21 and small amounts of taxes are paid on those inputs. The industry spends a small amount in foreign countries and pays factor incomes of 10 giving total inputs of 31 equal to the total of its output.

II. THE BASIC INPUT-OUTPUT SYSTEM

A. A simple input-output framework

2.1. The previous chapter looked at the national accounting framework and focussed attention on the treatment and role of an input-output table in this framework. The present chapter will concentrate on presenting an outline of basic input-output theory. This can be done most efficiently if the remainder of the social accounting matrix can be aggregated and simplified as much as possible.

2.2. It will be appropriate, therefore, to consider an economy which does not engage in foreign trade and in which there is no taxation. The various elements of final demand will be considered as a single vector and no subdivision of primary inputs will be made. In order to achieve maximum simplicity no distinction will be made between industries and commodities; it is assumed that industries, referred to here as production sectors, do not engage in any secondary production producing only their characteristic products.

Table 2.1. Highly simplified accounting framework

	Production sectors	Final demand	Totals
Production sectors	W (1)	f (2 - 5)	q
Primary inputs	y (16)		
Totals	q		

2.3. The accounting framework in this case is shown in Table 2.1 where the numbers in brackets refer to the cells in Table 1.3. The entries in Table 2.1, W, f, q and y are as defined in the Appendix I on notation. although it should be noted that, at this stage, no distinction is being made between industries and commodities. The input-output table is written as if it were a commodity x commodity table to preserve the conventional notation.

2.4. This input-output table which records the value of the sales and purchases between the production sectors in the economy is known as an input-output flow matrix. It is usual to write the typical entry in an input-output table in the form w_{ij}. This records the sales of sector i to sector j. Thus the entry $w_{1.2}$ will show the amount which sector 2 in the system purchases from sector 1.

2.5. It is thus possible to define the output of each production sector in terms of the amounts purchased by other production sectors - intermediate demand - and the amounts sold to final consumers, and for the whole economy we can write:

$$
\begin{aligned}
q_1 &= w_{1.1} + w_{1.2} + w_{1.3} + \dots + w_{1.n} + f_1 \\
q_2 &= w_{2.1} + w_{2.2} + w_{2.3} + \dots + w_{2.n} + f_2 \\
q_3 &= w_{3.1} + w_{3.2} + w_{3.3} + \dots + w_{3.n} + f_3 \\
&\vdots \\
q_n &= w_{n.1} + w_{n.2} + w_{n.3} + \dots + w_{n.n} + f_n
\end{aligned} \tag{2.1}
$$

2.6. The above set of structural equations express the input-output relations in terms of the entries in the flow matrix but it is often very useful to express the relations in terms of coefficients. A coefficient matrix records not the value of each transaction but the amount of each commodity purchased per unit of output of the purchasing sector. To form such a matrix each column of the flow matrix, W, must be divided by the total gross output of the purchasing sector. This coefficient matrix is denoted by A and the typical cell will be a_{ij} which can be defined as:

$$w_{ij} = a_{ij}\ q_j \tag{2.2}$$

In matrix notation this can be written as

$$W = A\hat{q}$$

or $$A = W\hat{q}^{-1} \tag{2.3}$$

2.7. The set of structural equations (2.1) can thus now be written in the following form:

$$\begin{aligned}
q_1 &= a_{1.1}\ q_1 + a_{1.2}\ q_2 + a_{1.3}\ q_3 + \dots + a_{1.n}\ q_n + f_1 \\
q_2 &= a_{2.1}\ q_1 + a_{2.2}\ q_2 + a_{2.3}\ q_3 + \dots + a_{2.n}\ q_n + f_2 \\
q_3 &= a_{3.1}\ q_1 + a_{3.2}\ q_2 + a_{3.3}\ q_3 + \dots + a_{3.n}\ q_n + f_3 \\
&\vdots \\
q_n &= a_{n.1}\ q_1 + a_{n.2}\ q_2 + a_{n.3}\ q_3 + \dots + a_{n.n}\ q_n + f_n
\end{aligned} \tag{2.4}$$

2.8. Here, each of the input-output relations is expressed in terms of a coefficient, a_{ij}, expressing the input as a proportion of the output of the purchasing sector and q_j, the output of that sector. This set of equations can be expressed in the form of matrices and vectors:

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} a_{1.1} & a_{1.2} & a_{1.3} & \dots & a_{1.n} \\ a_{2.1} & a_{2.2} & a_{2.3} & \dots & a_{2.n} \\ a_{3.1} & a_{3.2} & a_{3.3} & \dots & a_{3.n} \\ \vdots & & & & \vdots \\ a_{n.1} & a_{n.2} & a_{n.3} & \dots & a_{n.n} \end{bmatrix} \times \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \vdots \\ q_n \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_n \end{bmatrix}$$

The block of terms in 'a' are the entries in the matrix A and the above can be simplified to yield the basic input-output accounting equation:

$$q = Aq + f \tag{2.5}$$

2.9. When the equations are in this form they are much more suitable for any model-building or analysis. If the values of the coefficients are known and the level of final demand is known or assumed, it is possible to solve this set of equations to find the level of output of various commodities, q, as demonstrated in the next section.

2.10. If attention is turned to the columns rather than the rows of this simple system it is then possible to examine the purchases by each sector. Primary inputs can also be expressed in the form of coefficients, which can be denoted by $\bar{y}$, and since the primary inputs include the profit residual in addition to payments to labour, all expenditure by sectors will be accounted for. Reading down the column for a sector in the A matrix and through the $\bar{y}$ vector will show the proportions in which other products and primary inputs are used by the sector. This is known as its input structure.

2.11. In Tables 2.2 and 2.3 numerical illustrations are given of the points discussed in this section. Table 2.2. shows a simplified set of accounts similar to Table 2.1 distinguishing three production sectors and showing the input-output flow matrix.

Table 2.2. Input-output table and accounts

	Sector A	Sector B	Sector C	Final demand	Total output
Sector A		20	45	35	100
Sector B	30		30	140	200
Sector C		80		70	150
Primary nputs	70	100	75		
Total input	100	200	150		

Table 2.3. Input-output coefficient table

(a) Input per unit of output

	Sector A	Sector B	Sector C
Sector A		.10	.30
Sector B	.30		.20
Sector C		.40	

(b) Input per 1000 units output

	Sector A	Sector B	Sector C
Sector A		100	300
Sector B	300		
Sector C		400	200

2.12. In Table 2.3(a) this flow matrix is shown as a coefficient matrix and it can be seen that the entries in each column are obtained by dividing the entries in the flow matrix by the total output of the purchasing sector. It is sometimes felt desirable in presenting such tables to avoid several figures after the decimal point which occurs when there are small entries in cells in the flow matrix. Table 2.3(b) therefore shows inputs per 1000 units of output.

B. The inverse matrix

2.13. The input structures discussed in the previous section show what each sector requires to produce its output but it tells nothing about the further effects. The effect of the production of a motor-vehicle does not end with the steel, components and tyres purchased. It begins a long chain of production since each of the products purchased will, in their turn, require various inputs; production will be required in some industries not directly supplying the motor industry and the direct suppliers may be called on also to supply other industries.

2.14. It is thus possible to define two types of inputs. Direct inputs are those purchased by the industry under consideration; indirect inputs are those purchased by all industries in which production is required in order to supply inputs to the first industry. It can be seen from Table 2.3 that 1000 units of output in sector B requires 100 units of A and 400 units of C. These are the direct inputs. But the production of 100 units of A will require some units of B and the 400 units of C will require some A and some C, all of which in turn will generate other inputs from all sectors. If these indirect requirements can be traced through the system it is then possible to know the full impact of production in each of the sectors on the other sectors.

2.15. In this example the direct input requirements of 1000 units of B are shown in the second column of Table 2.4. The first set of indirect requirements will be 30 B required to produce 100 A; also 120 A and 80 B will be required as inputs to produce 400 C. These are calculated using Table 2.3 and are shown in the third column of Table 2.4. Various inputs will be needed to produce this output and this second round of indirect inputs is calculated and shown in the fourth column. The input requirements of successive rounds rapidly diminish and we can calculate the full impact of the demand for 1000 units of output of B. This is shown in the final column.

Table 2.4. Direct and indirect inputs

Producing sector	Initial output	Direct inputs	Indirect inputs						Total
			Round 1	Round 2	Round 3	Round 4	Round 5	Round n	
A		100	120	11	17	6	1		257
B	1000		110	36	12	8	1		1171
C		400		44	44	5	4		468

2.16. Similar calculations could be carried out for the other two sectors. The derivation of such results is complex and laborious by this method even with a 3-sector table. It is useful to return to the structural equations (2.4). The results above could be obtained by inserting the known coefficients, a_{ij} in these equations and solving them for q. In this case we have:

$$q_A = 0.10q_B + 0.30q_C$$
$$q_B = 0.20q_C + 1{,}000$$
$$q_C = 0.40q_B \qquad (2.6)$$

The solution of these equations will yield the total requirements of

1000 units of output of B and similar solutions can be made for 1000 units of A and C.

2.17. The most convenient way to obtain these results is to use the basic equation (2.5) and to solve this for q as follows:

$$q = Aq + f$$

$$q - Aq = f$$

$$(I - A)q = f$$

$$q = (I - A)^{-1}f \qquad (2.7)$$

The matrix, $(I - A)^{-1}$, is known as the Leontief inverse or the matrix multiplier (an analogy to the Keynesian multiplier).

2.18. If the matrix (I - A) is formed and inverted this example yields the matrix in Table 2.5.

Table 2.5. The inverse matrix, $(I - A)^{-1}$

	Sector A	Sector B	Sector C
Sector A	1.077	.257	.375
B	.351	1.171	.340
C	.141	.468	1.136

2.19. The columns of this matrix show the total input requirements, both direct and indirect, of one unit of output of the three sectors. The second column here is the same as the result obtained in Table 2.4. It will be noted that the entries in this matrix are considerably larger than those in Table 2.3(a), showing direct inputs; the differences between the two are the indirect inputs.

2.20. The calculation of the inverse of a matrix by hand is not recommended when the matrix is any larger than those in this example. It is similar to solving the set of structural equations (2.6) for each output level; one solution will yield the values for one column of the matrix.

Fortunately, computer programmes exist for matrix inversion and this method is recommended in practice. The principles involved are, however, best understood by the methods used here.

2.21. This inverse matrix $(I - A)^{-1}$, is fundamental to input-output analysis as it shows the full impact of the demand for the output of each sector on all the other sectors. With such a matrix it is possible to unravel the technological interdependence of the productive system and to trace the generation of demand from final consumers back throughout the system. It is then possible to calculate what output levels would be required to meet various postulated levels of final demand and consequently how output levels would be required to change to meet postulated changes in final demand, such as change in government expenditure.

2.22. In the same way as primary inputs were introduced when considering direct inputs in the coefficient matrix it is possible to extend the use of the inverse matrix to include primary inputs. Table 2.6 illustrates the primary input requirement of 1000 units of output by sector B.

Table 2.6. Primary input requirements

Sector	Direct & indirect inputs for 1000 units output of B (1)	Primary input coefficients (2)	Primary input requirements (3) = (1) x (2)
A	257	.7	180
B	1,171	.5	586
C	468	.5	234
			1,000

2.23. The outputs of products required were calculated in Table 2.5 and are shown in the first column of Table 2.6. Each of the producing sectors will require inputs of labour and capital to produce its required output. The primary input coefficients, $\bar{y}$, of each sector are calculated from Table 2.2 and shown in column 2 of Table 2.6. The third column is obtained as the product of the first two columns and shows the primary inputs required by each of the three sectors. It will be noticed that the total primary input requirement is 1000 units, exactly equal to the level of final demand assumed in the example. This identity in aggregate between final demand and primary input is, of course, required by national accounting principles.

2.24. Various and more detailed applications of the inverse matrix, which is central to input-output analysis, will be discussed and illustrated in Chapter V.

C. Prices and costs

2.25. It was noted earlier that columns of the input-output matrix toegether with primary inputs, accounted for all the expenditure of each sector. It follows that the price of each product can be built up from the price of each of its inputs combined with the relevant input coefficients. If, as previously, all inputs are considered including a coefficient for wage payments and the profit residual, the price of each product will be seen to depend on the price of other products and primary inputs. It is thus possible to write down a set of equations to show how price levels are determined and to solve these, providing that rates of payment to primary inputs and, of course, the A matrix, are known.

2.26. In the simple system in this chapter this will give

$$
\begin{aligned}
p_1 &= a_{1.1}\,p_1 + a_{2.1}\,p_2 + a_{3.1}\,p_3 + \dots + a_{n.1}\,p_n + \bar{y}_1 \\
p_2 &= a_{1.2}\,p_1 + a_{2.2}\,p_2 + a_{3.2}\,p_3 + \dots + a_{n.2}\,p_n + \bar{y}_2 \\
p_3 &= a_{1.3}\,p_1 + a_{3.3}\,p_2 + a_{3.3}\,p_3 + \dots + a_{n.3}\,p_n + \bar{y}_3 \\
&\vdots \\
p_n &= a_{1.n}\,p_1 + a_{2.n}\,p_2 + a_{3.n}\,p_3 + \dots + a_{n.n}\,p_n + \bar{y}_n \qquad (2.8)
\end{aligned}
$$

where p is the price per unit of output.

2.27. In this set of equations which represents the cost structure of each of the products, each of the input coefficients for a sector is multiplied by the price per unit of that input. If the values of the a_{ij} and the $\bar{y}_j$ were known it would be possible to solve this set of equations for p. However, as in the previous section, the solution is much simpler using matrix methods, particularly for anything but the smallest matrix. The price equations then become:

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_n \end{bmatrix} = \begin{bmatrix} a_{1.1} & a_{2.1} & a_{3.1} & + \cdots\cdots + & a_{n.1} \\ a_{1.2} & a_{2.2} & a_{3.2} & + \cdots\cdots + & a_{n.2} \\ a_{1.3} & a_{2.3} & a_{3.3} & + \cdots\cdots + & a_{n.3} \\ & & & & \vdots \\ a_{1.n} & a_{2.n} & a_{3.n} & + \cdots\cdots + & a_{n.n} \end{bmatrix} \times \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_n \end{bmatrix} + \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \\ \bar{y}_3 \\ \vdots \\ \bar{y}_n \end{bmatrix} \qquad (2.9)$$

2.28. The block of 'a' coefficients here is the coefficient matrix, A, with its rows and columns interchanged, i.e. the matrix is transposed and can be written as A'. The above set of equations can then be simply written:

$$p = A'p + \bar{y} \qquad (2.10)$$

which, when solved for p, gives:

$$p = (I - A')^{-1}\,\bar{y} \qquad (2.11)$$

2.29. It will be noticed that the inverse matrix here is very similar to the Leontief inverse in the previous section. The latter traces relationships 'backwards' through the productive system to yield estimates of output levels from assumed final demand. The matrix inverse in this section in effect traces relations 'forwards' through the productive system and builds up estimates of final product prices derived from the rates of payment to primary inputs. Thus the effects on price levels of changes in payments to primary inputs can be calculated. In Chapter V applications of this equation are discussed where primary inputs are disaggregated and indirect taxes included.

D. Basic assumptions

2.30. The use of input-output tables in analysis depends on two basic assumptions. The homogeneity assumption requires that each sector produces a single output with a single input structure and that there is no automatic substitution between the outputs of different sectors. The proportionality assumption states that the inputs into each sector are a linear function only of the level of output of that sector, i.e. that the amount of each kind of input absorbed by any particular sector varies in direct proportion to that sector's total output. These two assumptions are fundamental to the use of input-output tables and failure to meet these requirements can lead to inaccurate results. The problems which arise as a result of the heterogeneity of commodities are discussed in Chapter III, Section A.

2.31. In the pure theory of input-output the coefficients are regarded as relating to the physical quantities of commodities used in producing a given physical quantity of another commodity. In practice, however, almost all tables are prepared in money values. This is necessary because commodities are usually too heterogeneous to permit a purely physical measure and some, such as services, are simply not measurable in physical terms. It is important, however, that the coefficients in value terms should be interpreted in physical terms and treated as if they were technical coefficients. This requires that when input-output tables for different years are being compared they must be valued at constant prices. This is discussed further in Chapter III, Section N.

2.32. The coefficients in input-output analysis refer to the amount of each commodity used in the production of another and a distinction must be maintained between usage and purchase of commodities. The two will differ to the extent that the level of stocks of the input commodity changes during the time period to which the table refers. If purchases are greater (or less) than usage the difference will be recorded in final demand as investment (or disinvestment) in stocks - vector 4 in Table 1.2. In this way the recorded inputs will refer only to the amounts actually used in production and the technical nature of the coefficients will be preserved.

III. THE CONSTRUCTION TABLES

A. The statistical unit

3.1. It is possible to prescribe the ideal statistical units between which transactions in an input-output table should be recorded. This, however, will often remain a theoretical ideal. In practice, the form of the tables and the statistical units used will invariably depend on the nature of the available statistical data. Lack of adequate data may prevent the use of what may be considered the ideal statistical unit and some of the alternatives will be examined.

3.2. In selecting the statistical unit one of the most important aims should be to choose a unit which will meet the basic assumptions of homogeneity and proportionality referred to in the previous section. It is desirable to choose the unit so that the input coefficients will only change if there is a change in the techniques of production; for instance, if the unit consists of a mixed group of commodities (i.e. does not meet the homogeneity assumption) a change in the proportions in which these commodities are produced may result in a change in the recorded input coefficients even though there has been no change in the techniques of production of any of the commodities in the group. This failure to meet the proportionality assumption occurs because in this situation each recorded input coefficient is the weighted average of the input coefficients for each of the commodities in the group and will change if the weights, i.e. the output proportions, change. It will be obvious that the more detailed the tables and the finer the classification the less likely are such problems to arise. However, this might well result in prescribing very large tables which would be impracticable and very costly to produce and, further, adequate statistical data may not be available at such a fine degree of disaggregation.

3.3. It is generally recognised that the statistical unit may be one of the following: (a) a commodity group, (b) an establishment such as a farm, a mine or a factory or (c) an institution organising production in a branch of the economy such as an industrial enterprise or a government agency.

3.4. A grouping of commodities, if carefully chosen, would come close to meeting the assumptions and could prove to be a satisfactory classification. Many available industrial statistics provide the necessary information to permit an estimation of the destination of outputs to main purchasers of commodities. What is invariably lacking, however, is information on the input structure of commodities.

3.5. The establishment is a conventional accounting unit and consequently is able to provide information on inputs; production censuses in many countries take the establishment as the statistical unit. Details of inputs recorded by establishments, however, are often unable to distinguish between the inputs purchased for the production of particular commodities. Where the production by an establishment is concentrated entirely on one commodity or commodity group, this does not present any problems as far as the two basic assumptions are concerned. However, a number of establishments will be found with production in more than one commodity group and here the assumptions are unlikely to apply. Further, what is also lacking when dealing with an establishment with a mixed range of production is any data on the destination of its heterogeneous output, since output and sales statistics invariably relate to commodities.

3.6. The institution has similar benefits and poses similar problems to the establishment as a statistical unit but on a greater scale. An industrial enterprise or corporation may well embrace a number of establishments and may well produce a very diverse range of commodities. It is, however, likely to be a sophisticated source of statistical information. It will know amounts of various commodities it produces although not necessarily their destination. It will also record details of its purchases although, as in the case of the establishment, it may not be able to provide separate data on inputs into the various commodities it produces. Further, in a large organisation there will be a number of inputs, such as management and financial services, which are inputs into the enterprise and are common inputs to all commodities; it is usually very difficult to allocate such inputs between various commodities. Similar problems exist if the unit being considered is a government agency. Such institutions, although perhaps sophisticated in their accounting, are too large to be a suitable statistical unit for input-output tables.

3.7. It emerges from the above discussion of the various possible units that the nearest one can approach to meeting the two basic assumptions is to use the commodity as the statistical unit for output and sales data and the establishment as the unit for input data. The establishments will be classified to industries as discussed in Chapter I, Section C. Accordingly, it is usually appropriate to estimate the basic table, the absorption matrix, in the form of purchases of commodities by industries, i.e. as sub-matrix 1 in Table 1.2. If all establishments produced only one commodity (or commodity group) both sets of data would be on a commodity basis. However, problems do arise in the use of the tables when the inputs into establishments are used to produce several commodities. It is, however, possible to derive 'pure' tables from the basic absorption matrix which come sufficiently close to meeting the two basic assumptions. The problems and methods of dealing with such secondary production are discussed in the following section.

B. The treatment of secondary production

1. The nature of the problem

3.8. Many establishments produce only one commodity or group of commodities which is the characteristic product of the industry to which they are classified. However, as noted in the previous section and in Chapter I, Section C, some establishments produce other commodities which are not among the characteristic products of the industry to which they are classified. As a result in the make matrix, (matrix 7 in Table 1.2), industries are often recorded as producing several commodities. The amount of this subsidiary production varies between industries and is often not large, as can be seen in Table 1.3. However, it is the existence of such secondary production and lack of complete correspondence between industries and commodities which presents problems in deriving 'pure' input-output tables. These may be either commodity x commodity tables (recording the inputs of commodities into the production of commodities) or industry x industry tables. The relative merits of these two forms of tables are discussed in the next section.

3.9. In order to examine these techniques let us use a simplified accounting framework based on Table 1.2. The entries are shown in Table 3.1 and all are defined in the Appendix I on notation.

Table 3.1

	Commodities	Industries	Final demand	Total
Commodities		X	f	q
Industries	M			g
Primary inputs		y'		
Total	q'	g'		

3.10. From this accounting data various other matrices can be calculated as shown in Table 3.2. The derivation of some elements is not shown because various methods exist which yield different solutions.

Table 3.2

	Commodities	Industries	Final demand
Commodities	A $W = A\hat{q}$	$B = X\hat{g}^{-1}$	
Industries	$C = M'\hat{g}^{-1}$ $D = M\hat{q}^{-1}$	E $Z = E\hat{g}$	e
Primary inputs			

3.11. It was noted in the previous section that the basic input-output table, the absorption matrix, X, will be a commodity x industry table recording the inputs of commodities into industries. Most of these inputs are required to produce the characteristic product of the industry but some are required to produce its subsidiary products. The construction of commodity x commodity (A or W) or industry x industry tables (E or Z) from the basic data involves transfers of inputs and outputs between sectors. The transfer of outputs is a comparatively simple matter since the production of non-characteristic products will be recorded in the make matrix, M. The transfer of inputs, however, is not nearly so simple; as was noted in the previous section much of the basic data on inputs relates to inputs into industries and not to the particular commodities produced in those industries. It may be possible to obtain some data at a level below that of the establishment and in some cases engineering data may be available on inputs into certain commodities. But, as a rule, supplementary information of these kinds is incomplete and it is necessary to resort to some extent to mechanical means of making the transfers using mathematical methods.

3.12. Since it is generally harder to transfer inputs than it is to transfer outputs it will be useful first to examine a method which involves the transfer of outputs only; later, the various methods available for transferring inputs as well as outputs will be examined.

2. Transfer of outputs

3.13. The first step in this method is to convert the make matrix into a diagonal matrix by adding into the diagonal element, cell $M_{i.i}$, the off-diagonal elements in both row i and column i. This new diagonal element can be thought of either as the total production of commodity i plus secondary production by industry i or as the total production of industry i plus the secondary production of commodity i by other industries.

3.14. When this operation has been carried out for each row and column the matrix has been transformed into a diagonal matrix in which subsidiary production is duplicated since it appears in the diagonal element of the industry in which it is actually produced and also in the diagonal element of the industry in which it is the characteristic product.

3.15. The second step is to take all the off-diagonal elements in the make matrix, M, and add them to the corresponding elements in X. This means that each industry is deemed to acquire, and so record in its input structure, the output of its own characteristic commodity produced in other industries.[2/] Thus, if industry i produces 2 units of commodity j the value of the input in X of commodity j into industry i will be increased by 2 units.

3.16. This method is straightforward but it is difficult to say to what the resulting input-output relates. The resulting matrix cannot be said to relate exactly to either commodities or industries since as was explained above in paragraph 3.13, the transformation of the make matrix results in a diagonal matrix which is open to two interpretations, neither of which are pure industry or commodity. This situation arises because output of secondary products is transferred but the corresponding inputs are not.

3.17. A variation to the above method of transferring the output of secondary products can be noted. The method described above can be regarded as transferring outputs along the rows; the alternative method is to transfer outputs along the columns. In this case, if, as in the example in paragraph 3.15, industry i produces 2 units of commodity j this amount will be shown as a negative input of commodity j into industry i. This method suffers from the same disadvantages as the row transfer method and further introduces the possibility of negative cells in the adjusted input-output table.

3. Transfer of inputs

3.18. We shall now examine three methods which transfer both outputs and inputs in order to arrive at 'pure' industry or commodity tables. We shall show how it is possible to introduce considerable flexibility into what may seem at first to be a rigid, mathematical solution.

2/ This method is illustrated in the S.N.A. Paragraphs 3.19 through 3.23, on which the present description is based.

3.19. In order to estimate the input structure of commodity j from the known input structure of industry j (i.e. in order to estimate a column of the commodity x commodity table) it is necessary to deduct from the inputs of industry j those inputs which are required for the production by industry j of commodities other than its principal or characteristic product, commodity j. It is also necessary to add in the inputs required for the production of commodity j in other industries. Typically, no information is available as to the allocation of inputs in an establishment or industry between the various commodities produced and it is, therefore, necessary to make some assumption about these input structures in order to derive a commodity x commodity table.

3.20. When combining the information in the make and absorption matrices to estimate a 'pure' table, two basic assumptions are possible and these are generally referred to as the commodity technology and industry technology assumptions. The former assumes that a commodity has the same input structure in whichever industry it is produced. The industry technology assumption, on the other hand, assumes that all commodities produced by an industry are produced with the same input structure and thus commodities will have different input structures depending on the industry in which they are produced.

3.21. The explanation will be made clearer with the use of a simple numerical example where the basic data is:

X Absorption matrix

	Ind.		
Commod.	10	60	
	40	60	20
	20	30	60

M Make matrix

	Commod.		
Ind.	90	10	
		280	20
		10	190

Industry outputs, g = 100, 300, 200

Commodity outputs, q = 90, 300, 210

3.22. From these we derive the following:

$B = X\hat{g}^{-1}$ (x 1,000)

	Ind.		
Commod.	100	200	
	400	200	100
	200	100	300

D Market shares

	Commod.		
Ind.	1.000	.033	
		.934	.095
		.033	.905

C Product mix

	Ind.		
Commod.	0.90		
	0.10	0.94	0.05
		0.06	0.95

C^{-1}

	Commod.		
Ind.	1.111		
	-0.118	1.068	-0.056
	0.007	-0.068	1.056

3.23. Let us look first at the commodity technology assumption. In this case the inputs into industry j comprise the weighted average of the inputs into each of the commodities which it produces, the weights being the proportions in which industry j produces the various commodities. Thus $b_{1.2}$, the coefficient for input of commodity 1 into industry 2, can be written:

$$b_{1.2} = a_{1.1}\,c_{1.2} + a_{1.2}\,c_{2.2} + a_{1.3}\,c_{3.2}$$

or in general terms

$$b_{i.j} = a_{i.1}\,c_{1.j} + a_{i.2}\,c_{2.j} + a_{i.3}\,c_{3.j} \qquad (3.1)$$

3.24. There will be 9 (= 3 x 3) such equations in this example and it would be possible to write out all the equations and solve for a. This set of equations can be more conveniently expressed in matrix form:

$$\begin{bmatrix} b_{1.1} & b_{1.2} & b_{1.3} \\ b_{2.1} & b_{2.2} & b_{2.3} \\ b_{3.1} & b_{3.2} & b_{3.3} \end{bmatrix} = \begin{bmatrix} a_{1.1} & a_{1.2} & a_{1.3} \\ a_{2.1} & a_{2.2} & a_{2.3} \\ a_{3.1} & a_{3.2} & a_{3.3} \end{bmatrix} \times \begin{bmatrix} c_{1.1} & c_{1.2} & c_{1.3} \\ c_{2.1} & c_{2.2} & c_{2.3} \\ c_{3.1} & c_{3.2} & c_{3.3} \end{bmatrix}$$

$$B = AC \tag{3.2}$$

which in order to derive A from our basic matrices we can re-write as:

$$A_c = BC^{-1} \tag{3.3}$$

A_c denoting the commodity technology solution as opposed to the industry technology solution A_I.

3.25. Using the numerical example we obtain

$$A_c = \begin{bmatrix} 87 & 213 & -11 \\ 422 & 207 & 94 \\ 213 & 86 & 311 \end{bmatrix}$$

This can be better illustrated by calculating B from A_c and C. With a commodity technology assumption the inputs into industry j are the weighted average of the inputs into each of the commodities it produces, the weights being the industry's product mix. Thus, for instance,

$$b_{1.2} = (87, 213, -11) \times (0, 0.94, 0.06) = 200$$

3.26. If, on the other hand, we use an industry technology assumption the inputs into commodity j will be the weighted average of the inputs into each of the industries which produces commodity j and the weights will be the market shares of each industry in the production of commodity j. We thus would obtain as the general term

$$a_{ij} = b_{i.1}\, d_{1.j} + b_{i.2}\, d_{2.j} + b_{i.3}\, d_{3.j} \tag{3.4}$$

3.27. This set of nine equations can be expressed in matrix form:

$$\begin{bmatrix} a_{1.1} & a_{1.2} & a_{1.3} \\ a_{2.1} & a_{2.2} & a_{2.3} \\ a_{3.1} & a_{3.2} & a_{3.3} \end{bmatrix} = \begin{bmatrix} b_{1.1} & b_{1.2} & b_{1.3} \\ b_{2.1} & b_{2.2} & b_{2.3} \\ b_{3.1} & b_{3.2} & b_{3.3} \end{bmatrix} \times \begin{bmatrix} d_{1.1} & d_{1.2} & d_{1.3} \\ d_{2.1} & d_{2.2} & d_{2.3} \\ d_{3.1} & d_{3.2} & d_{3.3} \end{bmatrix} \tag{3.5}$$

which gives the industry technology solution:

$$A_I = BD \tag{3.6}$$

3.28. In our numerical example this solution yields

$$A_I = \begin{bmatrix} 100 & 190 & 19 \\ 400 & 203 & 110 \\ 200 & 110 & 281 \end{bmatrix}$$

Here, the entry $a_{1.2}$ is calculated from

$$(100,\ 200,\ 0) \times (0.033,\ 0.934,\ 0.033) = 190$$

The first column of A_I is the same as the first column of B because all commodity 1 is produced in industry 1. An entry appears at $a_{1.3}$ although $b_{1.3}$ was zero because some commodity 3 is produced in industry 2 and there is an input of 1 into 2 in B.

3.29. If it is desired to have an industry x industry table this can be obtained from the absorption and make matrices in a similar manner to that used for commodity x commodity tables and again two solutions are possible. Either matrix C or matrix D may be used as weights and the choice between them is dependent on making either a product-mix assumption or a market share assumption about the output structures in matrix M.

3.30. If we adopt a market share assumption, this implies that the shares of industries in the production of a particular commodity are stable. The input of industry i into industry j in matrix E will be made of the input of each commodity into industry j weighted by share of industry i in the production of each commodity. If we write E_M for an industry x industry matrix derived on a market-share assumption we have

$$E_M = DB \tag{3.7}$$

3.31. The numerical example will be:

$$E_M = \begin{bmatrix} 113 & 207 & 3 \\ 393 & 196 & 122 \\ 194 & 97 & 275 \end{bmatrix}$$

The cell $e_{1.2}$ is calculated from matrices D and B as

$$(1.000,\ 0.033,\ 0) \times (200,\ 200,\ 100) = 207$$

The entry $e_{1.3}$ is 3 although $b_{1.3}$ is zero. This arises because industry 3 uses some commodity 2 and some of this is produced in industry 1. This entry is

$$(1.000,\ 0.033,\ 0) \times (0,\ 100,\ 300) = 3$$

3.32. If we use matrix C with B to produce E we are using a product-mix assumption which is best explained by calculating B from C and E. The input of commodity i into industry j is the input of each industry into industry j weighted by the proportion which commodity i is of each industry's output - its product-mix and we have

$$B = CE_p$$

$$E_p = C^{-1}B \qquad (3.8)$$

3.33. The solution in our numerical example is

$$E_p = \begin{bmatrix} 111 & 222 & 0 \\ 404 & 184 & 90 \\ 185 & 94 & 310 \end{bmatrix}$$

As in the case of the commodity technology solution for A which also involves the inverse of C this is perhaps better illustrated by the calculation of B which in this case is $B = CE_p$. The input of commodity 2 into industry 1 is the input of each industry into industry 1 weighted by the proportion which commodity 2 is of each industry's output. Thus

$$b_{2.1} = (0.10,\ 0.94,\ 0.05) \times (111,\ 404,\ 185) = 400$$

3.34. The solutions for E in equations (3.7) and (3.8) are referred to respectively in the S.N.A. as industry technology and commodity technology assumptions, by analogy to the use of matrices D and C in the derivation of the A matrix.

3.35. The solutions given by equations (3.3) and (3.6) for commodity x commodity tables require an unnecessarily rigid approach. It is necessary to assume either that all subsidiary production has a commodity technology or that all subsidiary production has an industry technology. It seems reasonable to expect that some subsidiary production might fit a commodity technology assumption whilst for other elements of subsidiary production an industry technology assumption may be more appropriate. In general one would expect that most commodities have the same input structure wherever they are produced but, particularly where subsidiary production consists of by-products of an industrial process, the assumption of an industry technology may well be more appropriate in some cases.

3.36. This mixture of technology assumptions is incorporated in the S.N.A. (paragraphs 3.27 and 3.87 - 3.91) to give what can be called a hybrid technology solution. In order that various elements of production can be treated on different assumptions it is necessary to split the make matrix into two matrices, M_1 and M_2, where M_1 includes those elements of production for which a commodity technology assumption is deemed appropriate and M_2 includes those elements which are to be treated on an industry technology assumption.

3.37. The details of this form of solution and illustrations are contained in the S.N.A. and will not be repeated here. This method was successfully used to produce the 70-sector commodity x commodity tables for the U.K. for 1963. Compared with either the industry or commodity technology solutions the hybrid technology solution is not so simple. All cells in the make matrix have to be examined and allocated to either M_1 or M_2, depending on whether it is felt that their input structure is closer to that of the commodity group to which they belong or to that of the industry producing this cell of output. Further, the mechanical calculation calls for a somewhat more elaborate computer programme. On the other hand, this approach does prevent the need for using either of the two "extreme" solutions and introduces desirable flexibility.

3.38. In discussing the relative methods of the various solutions, the scale of the problem must be kept in perspective. If there were no secondary production the two simple technology assumptions would yield the same result. Differences only arise to the extent that there is secondary production; the amount of this in practice will depend partly on how production is organised in the economy, partly on the choice of statistical unit (e.g. the use of the institution rather than the establishment is likely to increase the amount of recorded secondary production) and partly on the size of the tables. More secondary production will generally be found in larger tables. This is because much secondary production will often be of commodities similar to the main production and when tables are aggregated these commodities may well be classified to the same group and the production will then appear as production of the characteristic product of the group in the diagonal of the make matrix. In the U.K. 1963 tables secondary production was 5% of all production in the 70-sector tables; this falls to 2.6% if the tables are aggregated to 13 sectors and to only 1.6% in the 6-sector table in Table 1.3.

3.39. Thus, in a typical 70-sector table the "problem" exists in connection with about 5% of industrial output. Further, in many of the cells where subsidiary production was quite high there were relatively similar input structures in both the main and subsidiary producing industries, so that the industry and commodity technology solutions yielded similar results. A comparison of the commodity x commodity tables calculated for the U.K. 1963 on each of the two simple assumptions showed that in the majority of cells differences were almost insignificant.[3/] It would seem, therefore, that many of the cells of subsidiary production in the make matrix can be treated fairly arbitrarily. This is because they are a small proportion of their industry and commodity output and/or the input structures of their industry and commodity are similar. There were, however, a number of cells where the choice of technology was important and where different assumptions led to quite large differences between the various matrices. It is because of the existence of such cells that a hybrid technology solution is useful since it allows some to be treated on a commodity technology assumption and some on an industry technology assumption.

C. Commodity or industry tables

3.40. The discussion of the appropriate statistical unit in Section A suggested that it was generally desirable in the construction of tables to make use of available data on outputs and sales of commodities and on input structures of establishments. The basic table, the absorption matrix, estimated directly from available statistics would thus be a commodity x

3/ See Armstrong, "Technology Assumptions in the Construction of U.K. Input-Output Tables", on which much of this section is based, in Aspects of Input-Output Analysis, edited by I.G. Stewart (Edinburgh University Press, 1972).

industry table. In the previous section various methods were outlined to show how it was possible to derive 'pure' tables from the absorption and make matrices. As these 'pure' tables may be either commodity x commodity or industry x industry tables it is appropriate to discuss the relative merits of the two forms.

3.41. If a commodity x commodity table was being used the basic equation would be as equation (2.5):

$$q = Aq + f \tag{3.9}$$

If, on the other hand, one was working with an industry x industry table the equation would be:

$$g = Eg + e \tag{3.10}$$

In both cases the input-output table could be calculated using any one of the three technology assumptions in the previous section.

3.42. Most applications of input-output analysis begin with a given final demand for commodities, f, and, by tracing the direct and indirect requirements of this level of final output throughout the productive system, calculate the corresponding levels of commodity outputs, q. For this purpose a commodity x commodity table and equation (3.9) is the more appropriate. Other applications proceed to examine the primary inputs into industries required for calculated output levels. This can be done more conveniently if industry x industry tables are being used but the same results can be obtained with little more difficulty if a commodity x commodity table is being used. The calculated commodity outputs can be transformed using the market share or product mix matrix into industry output levels (see previous section). This sequence of calculation would make an industry x industry table unnecessary.

3.43. In order to use an industry x industry table and equation (3.10) it is necessary to know the final demands for the outputs of industries, e. However, information on final demand is invariably available on a commodity rather than an industry basis and in order to estimate the final demand for industry outputs it is necessary to know the proportions in which industries produce the commodities entering each of the final demand vectors. It is possible that these proportions will not be the same for each one of the final demand vectors and such information is not usually available. A reasonable approximation would be to assume the same proportions throughout and use the coefficients of the market share matrix, D, to transform final demands for commodities into demands for industrial outputs, i.e. $e = Df$. A further problem arises, however, if equation (3.10) is being used to estimate output levels required to meet a postulated level of final demand for commodities, perhaps in a future year. It then becomes necessary to make an estimate of the industries' market shares, (i.e. to assume they are stable over time or to forecast how they will change) in order to obtain final demand for industry outputs.

3.44. It thus emerges that the commodity x commodity table will be found to be more suitable in most applications; demand is for a particular commodity or group of commodities and not for the mixed range of output of an industry and there is no need to transform the demand vectors from one unit to another. One should note, however, that such transformation is also necessary if commodity x commodity tables are used in an application which includes primary inputs, although due to the degree of similarity between primary input structures this may present less difficulty than the transformation of final demand in the case of industry x industry models.

3.45. Finally, it should be noted that differences between a commodity x commodity table and an industry x industry table will only arise to the extent that secondary production is recorded in the make matrix. Without secondary production the distinction between commodities and industries would disappear. The extent of secondary production is unlikely to be as great as in the example in the previous section and the differences between commodity and industry tables accordingly may be slight as a comparison of Tables 3.6 through 3.9 in the S.N.A. will show.

D. Rectangular tables

3.46. The input-output tables discussed in previous sections, whether commodity or industry tables, have all been square. The classification systems used have defined the same number of commodities as industries because each industry is defined in terms of its principal product. The absorption and make matrices and any tables derived from them are thus all square. There are, however, some advantages to be gained by abandoning this one-to-one correspondence between industries and their characteristic products. If this is done the characteristic product of each industry can be divided into several commodities. Hence the make and absorption matrices are rectangular and not square, but this does not prevent the use of such tables in input-output models. The Canadian input-output tables for 1961 distinguish in their published form 110 industries and 197 commodities; in their worksheet form there are 187 industries and 644 commodities. A very useful and lucid explanation of the uses of rectangular tables (and of input-output models, in general) is to be found in the descriptive volume published with these tables.[4/]

3.47. A system of rectangular tables permits a disaggregation of commodities without the need for a similar disaggregation of industries. Information on inputs into industries is often available in considerable product detail in production censuses and this information on many products need not be aggregated into commodity groups to the same extent as they would in a system of square tables. To attempt a similarly fine classification of establishments would present difficulties with the classification of multi-product establishments and in the allocation of common inputs among the many industries which a large enterprise would now cover. Rectangular tables, thus go at least part of the way to distinguishing the effects on the economy of demand for a larger number of commodities even if complete separation (i.e. a breakdown of the columns) is impracticable.

3.48. A rectangular table would be useful in a situation where a commodity group could be separated into two (or more) products, each of which, although having the same input structure, had a different supply or demand pattern. One might separate, for instance, two products from a group because the proportions of the supplies coming from imports were different.

4/ The Input-Output Structure of the Canadian Economy, 1961, Dominion Bureau of Statistics, Ottawa, 1969.

3.49. When one is working with rectangular tables there are limitations in the derivation of the commodity x commodity tables from the make and absorption matrices as described in Section B of this chapter. The commodity technology solution, ($A = BC^{-1}$) given in equation (3.3) cannot be used since C, like M, is a rectangular matrix and the inverse of a non-square matrix does not exist. Similarly, hybrid technology solutions are excluded and one must use an industry technology solution. This may generally be regarded as somewhat inferior to the commodity technology solution but this comparison should only be made between tables of the same size. With a rectangular table with, say, 300 commodities and 80 industries, the assumption of an industry technology means that one is assuming that each of an industry's, say, three principal products has the same input structure and this may be regarded as somewhat inferior to a commodity technology solution with a 300 x 300 table. As has already been said such detail is unlikely to be available, and an industry technology solution with rectangular tables is no less accurate than using a commodity technology assumption with an 80 x 80 table, in which case this industry's three principal products would be treated as a single product throughout.

3.50. It should be noted that rectangular tables only meet the homogeneity assumption (see Chapter II, Section D) if the product mix of industries is constant. Changes in the product mix may alter an industry's inputs thus causing a change in coefficients. This may present a problem if one is using the table to assess the effects of a new level of final demand; this will require new output levels which may well change the product mix and thus the input coefficients. If, however, the classification of industries and commodities has been suitably arranged the several principal products of the industry may have sufficiently similar input structures for this not to be a serious problem. Indeed, it is most unlikely that it will offset the gains to be derived from the use of the rectangular table distinguishing, as it does, many more commodities than one could reasonably hope to have in a square table.

3.51. A useful method of computing commodity output levels with rectangular tables is discussed in the Canadian volume mentioned earlier in this section. The normal solution would be to substitute the industry technology solution for A, the commodity x commodity matrix in equation (3.6) into the basic input-output equation (2.7) and obtain:

$$q = (I - BD)^{-1} f \qquad (3.11)$$

If we assume that we have 80 industries and 300 commodities matrix B will be 300 x 80 and D will be 80 x 300. Matrix BD therefore will be 300 x 300 and this solution requires the inverse of a 300 x 300 matrix which is a major computing task.

3.52. The alternative method adopted in the Canadian case is to solve first for industry outputs using an industry x industry matrix and then convert the industry outputs into commodity outputs. The accounting relation for industry outputs comparable to (2.5) for commodities has been given in equation (3.10) as:

$$g = Eg + e \qquad (3.12)$$

and introducing the solution for E in equation (3.7) we obtain:

$$g = (I - DB)^{-1} e \qquad (3.13)$$

The final demands for industry outputs can be converted to final demands for commodities using the market share matrix D, to give:

$$g = (I - DB)^{-1} Df \qquad (3.14)$$

3.53. The output of commodities normally written as in equation (2.5) can usefully be expressed as:

$$q = Bg + f \qquad (3.15)$$

and if the values for g obtained in (3.14) are substituted we obtain:

$$q = [I + B(I - DB)^{-1} D] f \qquad (3.16)$$

It can be shown that the matrix in square brackets in (3.16) is identical to the matrix $(I - BD)^{-1}$ in (3.11). The advantage of this method of calculation is that the size of matrix to be inverted is 80 x 80, i.e. (I - DB) in (3.14), which is preferable to the inversion in (3.11) and also that industry output levels are obtained in the process of calculation.

E. Size of tables

3.54. The ideal criteria for classification in input-output tables discussed in Section A of this chapter are that each sector represents a homogeneous product and a homogeneous set of inputs. This, however, cannot be strictly applied in practice. Even an apparently homogeneous commodity such as coal consists of various qualities of coal and the difference in prices paid by industrial and domestic users reflect different amounts of material inputs and of labour and capital resources used in the production of different qualities. Some aggregation of commodities is thus inevitable in the construction of tables and in considering the amount of aggregation and the size of the tables the homogeneity and proportionality assumptions must be kept in mind.

3.55. Until comparatively recently, the size of tables was often limited by the computing facilities available to perform the necessary calculations both in the estimation and in the application of the tables. Before the development of large-store high-speed computers the inversion of the matrix was a major problem. It can now be said that the lack of computing facilities will rarely prove to be a limiting factor as far as the size of the table is concerned. The limit to increasing the size of tables is now much more likely to be time and cost involved. It takes substantial time to produce national tables of a relatively modest size. Increasing the size of tables beyond these dimensions is likely to increase the time and the cost at least in proportion to the number of sectors.

3.56. In many countries there exists pressure from users or would-be users of tables for much larger tables than exist at present. Whilst there may well be gains for industrial and commercial users if tables were further disaggregated, experience suggests that for use in general policy models the gains to be expected from large tables may be relatively small. Highly disaggregated tables are a means of recording detailed information in a consistent manner but there will be no gain in input-output applications unless the disaggregated sectors differ from one another in either output or input structure.

3.57. Little will be lost if sectors are aggregated providing the two basic assumptions (homogeneity and proportionality) are used as references. Differences in the pattern of demand for two commodities suggest that they should not be aggregated, particularly if the pattern and growth of demand varies between sectors of final demand. When looking at the degree of similarity between input structure as a basis for aggregation, it is necessary to look beyond commodity input structures. Differences in primary input structure, such as different labour and capital ratios, must be considered and also differences in the rates of tax paid. In this latter context if the tables are being applied in the field of taxation one may require a classification and degree of aggregation quite different from other uses. Another factor which may be important is the source of supply; if two otherwise similar commodities are imported in different proportions it is desirable to keep them separate if considering any applications involving estimation of import requirements.

3.58. It is possible that in many cases the tables will be used in all the applications referred to in the previous paragraph. This would result in various 'ideal' classifications and, if all were followed, an excessively large number of sectors. Some compromise is therefore inevitable but care must be taken to avoid departing too far from the basic assumptions.

3.59. The limit to the size of table which is ultimately prepared is often set by the amount of statistical data available and the cost of mounting special production censuses to obtain more information. This cost is not limited to the central agency organising the inquiry; other users of tables, although often eager for larger tables, may complain about the cost to themselves of providing the extra information - 'the burden of form-filling'.

3.60. It must also be remembered that the application of larger tables requires more resources than the smaller tables. This can only partly be met by modern methods of computation for there remains much that requires trained manpower, e.g. the projection of input coefficients and future patterns of demand. The work-load involved here will increase at least in proportion to the number of sectors and may, as suggested earlier, not result in much gain in general policy models compared with smaller tables.

3.61. There remain, obviously, applications, particularly by industrial and commercial users, where very large tables would be desirable. A possible solution may be for the government agency to concentrate its resources on producing a table which is sufficiently large for most general applications, distinguishing, say, 50 to 100 sectors and for other organisations to devot their resources and specialist knowledge to disaggregate those sectors of the table in which they are particularly involved. An agency representing the construction industry, for instance, could disaggregate the construction sector into a number of different types of building activity, e.g. houses, flats, schools, offices, hospitals, roads etc., recording for each the pattern of demand and input structure. The inputs into each of these sectors could also be further disaggregated to show in greater detail those products used by the construction industry. Such a combination of resources and effort by government statistical offices and specialist organisations could result in large areas of the economy being covered in considerable detail in the input-output framework.

F. Valuation

3.62. It is important in input-output as in national accounting that all transactions are valued in the same way. The purchase of one unit of a particular commodity by any buyer is assumed in input-output analysis to stimulate demand to the same extent. This will not happen if the valuations are different for different buyers. There are two main problems in ensuring uniform valuation. Firstly, where taxes are levied on outputs purchases may be valued either including or excluding the tax, i.e.

either at producers' values or at basic values. Secondly, there is the effect of trade and transport margins and the choice between producers' values and purchasers' values.

1. Basic values of producers' values

3.63. The sales of commodities to various classes of purchaser are shown in Table 1.2 in matrices 1 through 6. These transactions are valued at basic values. The taxes which are levied on these commodities are shown in cells 7 through 12. If the cells 1 through 6 are added to the corresponding cells in the commodity tax row the transactions will be valued at producers' values. For instance, in Table 1.3 total intermediate demand by the metals and machinery industry is 70 at basic values and 71 at producers' values; there is a much greater difference between the two valuations in the case of personal consumption, 171 at basic values and 197 at producers' values. Although not shown as such in Tables 1.2 and 1.3 each of the cells 7 through 12 can be of the same dimension as the corresponding cells in the commodity row. Cell 7, for instance, can be a matrix of the same size as the absorption matrix and the amount of tax (or subsidy) attached to each of the purchases in the absorption matrix will be shown in the corresponding cell in matrix 7.

3.64. It is thus possible to work with matrices 1 through 6 and to estimate the impact of various sectors of final demand on commodity outputs at basic values. Alternatively, it would be possible to combine matrices 1 and 7 to give an input-output table at producers' values and to similarly combine the final demand vectors and estimate the impact on commodity outputs at producers' values. In this latter case, if the rates of taxation were the same for all buyers, the resulting vector of commodity outputs could be adjusted to basic values by deducting taxes, the rate of which is known. In practice, however, such uniformity in the rates of taxation is not always found and different results would be obtained in this case, depending on which basis of valuation was being used.

3.65. Differences in valuation due to different rates of tax may occur between final and intermediate buyers or between the various classes of final buyers. Precise examples obviously depend on the exact nature of a national taxation system. One may find, for instance, that certain appliances, such as typewriters or refrigerators, are taxed when sold to personal consumers but are untaxed when sold for business use. High rates of tax may be levied on cigarettes when sold to domestic consumers but the tax may not be charged on export sales. The taxation system may distinguish between personal consumption and capital formation and a commodity (e.g. a motor vehicle) may be taxed in the case of the former but not the latter. Similar distinctions might be made between personal consumption and government consumption.

3.66. Differences in recorded rates of tax will also occur if the "commodity" consists of several products which are taxed at different rates and which are bought in varying proportions by different purchasers. For instance, if one of the commodities in the classification was petroleum products, this would include gasoline which may be highly taxed and fuel oil which may be taxed at a lower rate. One pound's worth of petroleum products will represent a much smaller quantity in the case of personal consumption (mainly gasoline) than in the case of industrial uses (mainly fuel oil) and there will be similar discrepancies within intermediate demand between purchases by the road transport industry and by other industries.

3.67. In the examples in the two previous paragraphs one pound's worth of purchases valued at producers' values will represent different amounts at basic values for the sectors taxed at different rates. The inaccurate results which may arise using producers' values and the importance of using basic values can be illustrated using the example shown in Table 3.12 of the S.N.A. which is reproduced here as Table 3.3.

3.68. Two industry x industry coefficient tables in 13 sectors for the U.K. in 1960 were used, one at basic values and one at producers' values, and the inverse matrices of each were calculated. (In the notation of Chapter III, Section B $(I - E_M)^{-1}$ was formed). The columns of the two matrices, showing the direct and indirect requirements of £1,000 of output of the industry group, food, beverages and tobacco are shown in columns 1 and 4 of Table 3.3. This product group is chosen because of the considerable difference in the tax on tobacco between domestic and foreign consumers. All production is subject to a high rate of tax but there is a drawback on export sales. This means that a given expenditure on exports

Table 3.3. Direct and indirect requirements at basic values per 1,000 units of final demand at producers' values for food, beverages and tobacco

	From inverse matrix at basic values			From inverse matrix at producers' values		
	Column for industry 3	Requirements at basic values		Column for industry 3	Average ratio of basic to producers' values	Requirements at basic values for any final demand
		Domestic final demand	Export demand			
	(1)	(2)	(3)	(4)	(5)	(6)
1. Agriculture	221	193	365	181	1.105	198
2. Mining	23	20	38	20	0.991	20
3. Food, beverages, tobacco	1193	1040	1971	1176	0.915	1076
4. Textiles, clothing	17	15	28	15	0.966	14
5. Chemicals, oil	115	100	190	103	0.980	101
6. Basic metals	26	23	43	22	0.997	22
7. Metals and machinery	75	65	124	67	0.961	64
8. Other manufactures	77	67	127	69	0.973	67
9. Utilities	21	18	35	19	0.973	18
10. Construction	19	17	31	17	0.999	17
11. Transport	60	52	99	54	0.990	53
12. Distribution	35	31	58	32	0.980	31
13. Services	57	50	94	52	0.894	46

of this product group buys more goods and includes less tax than a corresponding expenditure on the domestic market. In this particular case £1 of demand at producers' values represents £0.872 of domestic final demand at basic values (due to tax on beverages) and £1.652 of export demand at basic values (due to the 'negative' tax on tobacco). By multiplying column 1 by 0.872 and by 1.652 we obtain the requirements at basic values of £1,000 of domestic final demand (column 2) and of £1,000 of export demand (column 3). The figures in these two columns show the marked difference in the impact of the same amount of expenditure by these two sectors.

3.69. If we work entirely at producers' values (i.e. with our matrices 1 through 6 combined with matrices 7 through 12) no such differential effect will be observed. In order to obtain requirements at basic values we must remove from the figures the amounts of taxes included. The average ratio of basic to producers' values for each industry is given in column 5 and thus we obtain column 6.

3.70. It will be noticed that column 2 is similar to column 6. The latter is, in fact, the weighted average of column 2 where the ratio is 0.872 and column 3 where the ratio is 1.652 but because domestic demand is much larger than export demand the weighted average is 0.915; hence the similarity between columns 2 and 6. It can be seen from this example that similar results would be obtained on either basis of valuation if the tax ratios were similar for all sectors of demand and that even if there are differences in the ratios, if one sector is dominant, similar results will be obtained for that sector on either basis. For the other sectors (e.g. export demand in Table 3.3) very misleading results will be obtained. In general, if there are differences in the ratio of producers' to basic values a matrix calculated at producers' values should not be used.

2. Producers' values or purchasers' values

3.71. Any transaction may be valued at either the price received by the producer or the price paid by the purchaser; the difference between the two is composed of wholesale and/or retail distributive mark-ups and transport costs, i.e. trade and transport margins. Input-output tables (and the associated vectors of final demand) may, therefore, be valued at either producers' or purchasers' values, depending on how trade and transport margins are treated. In this section it will be assumed that there are no indirect taxes and a choice will be made simply between producers' and purchasers' values in order to demonstrate the correct treatment of margins.

3.72. In the last section it was shown how differences between various classes of buyers in the rates of taxation on commodities could result in inaccurate analysis and thus a recommendation to use basic values. In the same way differences between classes of buyers in trade and transport margins will result in purchasers' values giving inaccurate results compared with producers' values. It is important here also to ensure uniform valuation i.e. that one pound's worth of expenditure on a particular commodity will represent, in the case of each purchaser, the same demands on the productive system. This will not be achieved under a system of purchasers' values if there are differences in margins.

3.73. Such differences may arise in a number of ways. Within intermediate demand some firms or industries may buy a product direct from manufacturers, whilst others buy from a wholesaler or merchant, thus incurring distributive margins. A major difference occurs between domestic final buyers and industrial buyers in that the former will often pay retail margins and again there may be differences in margins between sales to domestic purchasers and exports.

3.74. The various methods of treating distribution and transport margins are illustrated in Tables 3.4 to 3.8. In this hypothetical example only two commodity and industry groups are considered in addition to the group covering distribution and transport. The make and absorption matrices and a single category of final demand are shown.

3.75. In Table 3.4 all transactions are valued at purchasers' values. There are margins of 40 on the sales of commodity 1 and 50 on sales of commodity 2 and these amounts are shown as inputs of distribution and transport into the two industries. In Table 3.5 all sales are recorded at producers' values and the margins are shown in a separate account. Here it can be seen that the final demand of 100 for commodity 1 at purchasers' values is made up of 65 of commodity 1 at producers' values plus transport costs and distribution margins of 35. This matrix of margins is aggregated into a single row in Table 3.6 which is the usual form taken by a set of accounts at producers' values; this row represents the margins on all purchases by each sector. The structure of final demand can be seen to be

Table 3.4. Purchasers' values

		Commodity 1	Commodity 2	Commodity D.T.	Industry 1	Industry 2	Industry D.T.	Final demand	Totals
Commodity	1				20	50	10	100	180
	2				100	20	10	150	280
	D.T.				40	50	-	-	90
Industry	1	180	0	-					
	2		280	-					
	D.T.	-	-	90					
Totals		180	280	90					

Table 3.5. Allocation of margins

		Industry 1	Industry 2	Industry D.T.	Final demand	Totals
Commodity	1	18	47	10	65	140
	2	91	20	9	110	230
		-	-	-	-	-
Margins on commodity	1	2	3	0	35	40
	2	9	0	1	40	50
		-	-	-	-	-

Table 3.6. Producers' values

		Commodity 1	Commodity 2	Commodity D.T.	Industry 1	Industry 2	Industry D.T.	Final demand	Totals
Commodity	1				18	47	10	65	140
	2				91	20	9	110	230
	D.T.				11	3	1	75	90
Industry	1	140	0	0					
	2	0	230	0					
	D.T.	0	0	90					
Totals		140	230	90					

Table 3.7. Producers' values with separation of distributive and transport margins

		Commodity 1	Commodity 2	Commodity D	Commodity T	Industry 1	Industry 2	Industry D	Industry T	Final demand	Totals
Commodity	1					18	47	8	2	65	140
	2					91	20	2	7	110	230
	D					9	2	0	1	58	70
	T					2	1	0	0	17	20
Industry	1	140	0	0	0						
	2	0	230	0	0						
	D	0	0	70	0						
	T	0	0	0	20						
Totals		140	230	70	20						

Table 3.8. Producers' values - alternative treatment of transport margins

		Commodity 1	Commodity 2	Commodity D	Commodity T	Industry 1	Industry 2	Industry D	Industry T	Final demand	Totals
Commodity	1					18	47	8	2	65	140
	2					91	20	2	7	110	230
	D					9	2	20	1	58	90
	T					2	1	0	0	17	20
Industry	1	140	0	0	0						
	2	0	230	0	0						
	D	0	0	90	0						
	T	0	0	0	20						
Totals		140	230	90	20						

Note: In Tables 3.4 through 3.8 a zero (0) indicates a nil entry in a particular cell and a dash (-) indicates that there can be no entry in that cell in the conceptual treatment being used.

65 of commodity 1 and 110 of commodity 2 at producers' prices plus 75 margins.

3.76. It is often felt desirable to show distribution and transport margins as two separate entries because their input structures are different and many official industrial classifications place the activities in different groups. This is done in the example in Table 3.7. It may be found that the necessary data is not readily available to permit this disaggregation of the margins across all purchasers, although the total margins paid by each purchaser may be estimated. In this case the treatment in Table 3.8 is possible. Here all margins are treated as distribution margins and the distribution industry is shown as purchasing 20 units of transport and its output and sales are increased by 20 compared with Table 3.7.

3.77. It should be noted here that the gross output of the distribution and transport industries should not include the values of the goods handled. Gross output is defined as the value of the mark-up, i.e. the difference between the selling and buying values in the case of distribution or the charge made for transportation. If the goods handled were included in the outputs and inputs of these industries the homogeneity of those sectors, and thus some of the usefulness of the table, would be lost.

3.78. Many available statistics record transactions at the price paid by the purchaser and thus tables at purchasers' values may be more easily prepared. It may not be easy to obtain the detailed information necessary for a matrix of margins as shown in Table 3.5 but attempts should be made to obtain and estimate sufficient data to permit a table at producers' values to be constructed as these are much to be preferred to tables at purchasers' values.

3.79. To conclude this section, the main advantages of producers' values compared with purchasers' values will be considered. Firstly, a system of producers' values will give the uniformity of valuation which was shown in the last sub-section to be important and which is unlikely to occur with purchasers' values. Secondly, under a system of purchasers' values the total output of each sector which is the basis for computing input coefficients includes the marketing costs incurred in each delivery of that sector's output. These marketing costs may well vary as the distribution of the sector's output changes and thus lead to variations in the recorded value of total output even if actual production is unchanged. This means further that coefficients estimated in the base year are unlikely to be stable. Thirdly, under a system of purchasers' values, all marketing costs are counted twice - in the value of output of the producing industry and as inputs to that industry from the distribution and transport industries. Under a system of producers' prices marketing costs are counted only once and duplication does not occur.

3. Valuation: Conclusion

3.80. The two previous sub-sections have discussed the relative merits of each of two pairs of methods of valuation. The first of the two sections concluded that in the treatment of indirect taxes valuation at basic values was preferable to producers' values but did not consider problems raised by distributive margins. The second section, assumed no indirect taxation, and discussed the treatment of margins. In practice, however, these two problems cannot be considered, nor can decisions on methods of valuation be made, in isolation. Purchasers' values differ from basic values by the amount of tax and by the amount of margins. Producers' values differ from purchasers' values by the amount of margins and from basic values by the amount of tax. The second sub-section recommends that margins be shown separately and transactions valued at producers' prices but does allow for any tax. When tax is considered the final recommendation must be the use of basic values, i.e. with both the tax and the margin component of each transaction shown separately from the commodity component. Finally, it should be remembered that if taxes are treated as in Table 1.2 and margins as Table 3.5 it is possible to aggregate the system of accounts in basic values to systems in either producers' or purchasers' values, if so desired.

G. Intra-industry transactions

3.81. It is the practice in some countries to exclude intra-industry transactions so that all cells in the principal diagonal of the input-output table are left blank. In this case the value of intra-industry transactions is deducted from gross output which is thus measured "free from duplication". If the sales from one establishment to another in the same industry are excluded gross output is then independent of the structure and organisation of the industry and of the number of statistical reporting units, usually establishments. If two establishments in an industry were amalgamated any transactions which previously took place between them would no longer be recorded and the statistics available would show a change in gross output and in input structure when the level and method of production is in fact unchanged. The situation may well arise, for instance, where a motor vehicle manufacturing firm gains control of motor component manufacturer and the production is integrated into a single establishment. Both of these firms would probably be classified to the same industrial group in the table and when this integration takes place the output and input of components will no longer be recorded and the output level of the industry will appear to have fallen. If, however, these intra-industry sales had been excluded from the original recording scheme, this problem would not have arisen.

3.82. A second reason for excluding the diagonal entry is that in some countries the organisational structure of the very large enterprise may prevent it from being able to supply data on outputs and inputs of individual establishments. It may collect this data for a production division of several establishments. Where these establishments produce the same final product this presents no problem but if one of these establishments supplies another in the division the necessary data may not be available to complete the intra-industry cell.

3.83. On the other hand, if the intra-industry transactions are included then the statistical record of the economy is complete and figures of gross output will then be more comparable with data on employment, capital and primary inputs used in any productivity studies. It should also be noted that the argument about the size of intra-industry transactions being affected by the structure of the industry can equally well be applied to the recording of other inputs. The motor vehicle manufacturer referred to earlier may integrate his production with that of the tyre manufacturer and unless special attention is taken the recorded input of rubber goods may be reduced as a result of this organisational change.

3.84. The main reason why intra-industry transactions should be included is that what is classed as an intra-industry transaction is dependent on the level of aggregation of the tables. If the manufacture of flour and of bread are both classed to, say, the cereal products industry, sales of flour by millers to bakers would be omitted if the diagonal entry were being excluded. If, however, the manufacture of flour and bread were treated as separate industries in a more disaggregated table, it would then be necessary to include these sales of flour. The only transactions to be omitted in the disaggregated table would be any between the millers themselves or between the bakers themselves. Thus, both the structure of industries' inputs and outputs and the size of gross outputs will be dependent on the level of aggregation if one attempts to exclude intra-industry transactions.

3.85. In order that all the transactions in the productive system are recorded, it would seem desirable to include the diagonal entries in input-output tables. If any particular user wishes to work with intra-industry sales excluded they are then free to omit the diagonal entries. The balance of preceding argument suggests, however, that it is preferable, in general, to include intra-industry transactions.

H. Treatment of imports

3.86. The treatment of imports in input-output tables varies considerably between countries and national practices are summarised in Appendix II. The treatments vary in the amount of statistical data they require and accordingly some have their appeal from this standpoint but, as is often the case, the simplest treatment is not always the most useful in applications of the tables. When an input-output table is being used to assess the output of commodities required to meet a given level of final demand, it is useful to be able to extend the analysis to determine the proportions in which the required supplies of the various commodity will be produced domestically or imported. This assessment of import requirements enables one to assess the implications for the balance of trade of a postulated set of final demands. In many economies imports represent a significant share of total supplies and balance of payments problems often restrict economic growth; it is, therefore, important to be able to analyse import requirements accurately in input-output models.

3.87. In the following sections of this chapter four alternative treatments of imports will be outlined and their relative merits discussed. The application of these treatments to assess import requirements is illustrated in Chapter V, Section C. Certain imports, namely expenditure in foreign countries, are treated in all four variants as direct transactions between the sector involved and the rest of the world as explained in paragraph 1.12 and will not be further discussed. Before discussing the various treatments of imports it is necessary to consider the question of the valuation of imports.

1. Valuation of imports

3.88. The importance of the uniformity of valuation of transactions was emphasised in Chapter III, Section F. This applies equally when one is considering supplies of commodities as when one is considering purchases of commodities. Domestically produced commodities will be valued at basic values, i.e. excluding any commodity taxes and distribution and transport margins. The valuation for imports which corresponds most closely conceptually to that used for domestic products is the value of import goods at the border of the importing country - which may be termed the domestic border or port value. This will be the c.i.f. value of imports plus protective import duties. The c.i.f. value of imports consists of three items:- (a) the value of goods when leaving the exporting country, (b) freight charges to the domestic port of entry, and (c) insurance charges.

3.89. It is appropriate to add to the c.i.f. value only those import duties which can be regarded as protective duties, i.e. duties levied in order to raise the cost of the import in relation to the corresponding domestic cost with the aim of putting the domestic and foreign producers in an equally competitive position. Other import duties aim mainly at raising revenue or discouraging consumption and are often matched by a corresponding excise duty on the domestically-produced commodity. Protective duties may be small (as in the case of the U.K. example in Table 1.3) and may present problems of definition but an attempt should be made in the interests of uniformity of valuation to enter them in cell 14 in Table 1.2. Other duties on imports which are comparable in their effect to taxes or excise duties on domestic goods will appear with them in cells 7 through 12.

3.90. This treatment of imports will ensure that they are valued in a manner comparable to the ex-factory pre-tax valuation used for domestic goods. Any distributive or transport margins involved in transferring imported goods from their port of entry to purchasers should be treated in the same way as similar margins on the transfer of domestic goods from the producer to the purchaser.

3.91. A problem arises as a result of the use of domestic border or port values for imports which require rather arbitrary treatment. The sum of the c.i.f. values of imports will overstate the value of imports on a balance of payments concept if any of the freight and insurance activities are carried out by domestic producers. For instance, in the case of the U.K. example used in Table 1.3 and in this section, the value of insurance and freight on imports was about £500m, of which £300m was supplied by U.K. producers and £200m by foreign producers. The total c.i.f. value of imports thus exceeded the balance of payments value by £300m and adjustment to the accounts was required. This amount can be entered in the import of commodities as negative values for transport and services and in the input-output table as negative inputs into the transport and services industries (to be set against other positive values in estimating the total value of those cells).

3.92. An alternative treatment is to enter the value of these freight and insurance services provided by domestic producers as an export of those commodities. This second treatment is probably preferable and is recommended by the S.N.A. (paragraph 6.137). Although it involves double-counting of these services (as part of the total value of imports and as part of domestic output) it avoids the possible appearance of a negative entry for transport when the import row is disaggregated.

2. Variant A: Imports classified by commodity

3.93. The treatment of imports in Chapter I is considered here as the first of the four variants. Table 3.9 reproduces the relevant parts of this accounting framework. All imports are given a commodity classification and are combined with production by domestic industries and these totals of commodity supplies are allocated to various purchasers. In this variant no information is required on the origin (domestic or foreign) of commodities purchased by each sector which makes such tables easier to construct than others and this is their main advantage.

3.94. The principal disadvantage of this variant is that the impact of a certain level of final demand on domestic production and imports cannot be assessed accurately. One is forced, in the absence of other information, to assume that each sector of demand requires imports in the same proportion to domestic supplies. Purchases by any sector of demand may generate a demand for imports in two ways. Firstly, through the purchase of imported goods (except in the case of export demand) and secondly imports may be required as direct or indirect inputs into the domestic supplying industry. Even if there is a considerable degree of disaggregation in the tables it will not be possible to obtain homogeneous "commodities" and thus it is quite likely that the import requirements of demand for a given commodity will vary between sectors. In this case, the use of average import ratios for each commodity will lead to inaccurate results.

3. Variant B: Imports classified by purchaser

3.95. In this variant no commodity classification of imports is shown and imports are shown in a single row entering into each of the sectors of demand. In Table 3.10 we see that of the 51 of total imports which entered the commodity account in Table 3.9, imports of 36 are purchased by industries, 13 by private consumers and 2 by capital formation. In a disaggregated table the intermediate demand of 36 would, of course, be allocated across the various purchasing industries.

3.96. This variant, which is quite often used, presents somewhat more information than that in Table 3.9 and if some additional data on the commodity classification of imports into final demand sectors is available it is then possible to assess the import requirements of postulated vectors of final demand. The disadvantage of this approach is that it excludes imports from the input-output table and conceals the nature of the imports, information which is very useful if some of the imports compete with domestically produced goods. If an import is a substitute for a domestic product then an increase in the purchases of the domestic product may occur at the expense of the import (or vice versa) thus causing a considerable change in the domestic input coefficient.

3.97. It is very likely that this variant will require more statistical information than it actually reveals. It is unlikely that available statistics will record the total imports entering each industry and sector of final demand. It will be necessary to estimate the totals of imports (required for the last row of the table) by considering the split between imports and domestic production in the input cells in each column. If this amount of information can be made available it seems desirable to use variant D considered below in sub-section 5.

4. Variant C: Complementary and competitive imports

3.98. In this treatment imports are divided into two categories and the supplies and purchases of the two categories are shown separately. Competitive commodities are those for which there is a domestic industry and which may, therefore, be either home produced or imported. Complementary commodities, on the other hand, are those for which no domestic industry exists, so that if they are needed they can be ontained only as imports. In Table 3.11 the commodity account is split into these two components and in this example we see that total competitive imports amounted to 41 and complementary imports were 10. Competitive imports are combined with domestic production and allocated to purchasers and are thus treated as were all imports in Variant A. Complementary imports are separately allocated to different purchasers - 7 to industries and 3 to personal consumption. In this example of the U.K. the complementary commodities included tobacco, crude oil, tea and coffee and non-ferrous metals and wood-pulp.

Table 3.9. Variant A: Imports classified by commodity

	Commod.	Commod. tax	Ind.	Consumption: Personal	Consumption: Govt.	Capital formation: Stocks	Capital formation: Fixed	World	Totals
Commodities			236	171	22	2	48	56	535
Commodity taxes			6	26	1	0	1	-0	34
Industry	483								483
Indirect taxes	1	34	-						
Factor incomes			237	4	27			4	
World	51		4	3	2	-	-		60
Totals	535	34	483	204	52	2	49	60	

Table 3.10. Variant B: Imports classified by purchaser

	Commod.	Commod. tax	Ind.	Consumption: Personal	Consumption: Govt.	Capital formation: Stocks	Capital formation: Fixed	World	Totals
Commodities			199	158	22	2	46	56	483
Commodity taxes			6	26	1	0	1	-0	34
Industry	483								483
Indirect taxes	-	34	1	0	0	0	0		
Factor incomes			237	4	27			4	
World	-		4) 36) 40	3) 13) 16	2) 0) 2	0	2		60
Totals	483	34	483	204	52	2	49	60	

Table 3.11. Variant C: Competitive and complementary imports

	Compet. commod.	Compl. commod.	Commod. tax	Ind.	Consumption: Personal	Consumption: Govt.	Capital formation: Stocks	Capital formation: Fixed	World	Totals
Competitive commodities				229	168	22	2	48	56	525
Complementary commodities				7	3	0	0	0	-	10
Commodity taxes				6	26	1	0	1	0	34
Industry	483									483
Indirect taxes	1	0	34	-	-	-	-	-		
Factor incomes				237	4	27			4	
World	41	10		4	3	2	-	-		60
Totals	525	10	34	483	204	52	2	49	60	

Table 3.12. Variant D: Separate entries for imported and domestic products

	Commod.	Commod. tax	Ind.	Consumption: Personal	Consumption: Govt.	Capital formation: Stocks	Capital formation: Fixed	World	Totals
Commodities -domestic			199	158	22	2	46	56	483))535
Commodities - imported			37	13	0	0	2	-	52)
Commodity taxes			6	26	1	0	1	0	34
Industries	483								483
Indirect taxes	1	34	-	-	-	-	-		
Factor incomes			237	4	27			4	
World	51		4	3	2	-	-		60
Totals	535	34	483	204	52	2	49	60	

Note: (i) These tables are based on Table 1.1 relating to the U.K. in 1963.

(ii) A dash (-) in a cell indicates that in that variant there will be no entry although in others there may be.

3.99. This distinction is important in input-output analysis because if complementary commodities are needed they must be imported; they cannot be obtained at home however much domestic production may be stimulated. If this distinction is not made the analysis will work as if any commodity would be obtained in given proportions from domestic production and imports and it would follow that a country with, say, any kind of mining industry would be deemed to meet at least part of the demand for all mining products at home even if some of these products were not in fact produced in the country. Due to differences in final product-mix different vectors of final demand will imply different proportions of complementary imports. If the country mines coal but not oil, the direct and indirect effects on the productive system of a final demand vector implying a given amount of mining products will depend on the proportions in which this required amount is made up of coal and oil. If coal is classed as a competitive commodity and oil as a complementary commodity then the effects on domestic production and imports can be more accurately assessed.

3.100. This treatment then is superior to variant A and, in fact, does not require much more statistical information. It will often be found that complementary imports will consist of a small number of commodities where the value of imports is quite high. In the U.K. example five clearly defined commodities accounted for all the complementary import account - an amount equal to about 20% of all imports. It is thus necessary merely to allocate these homogenous products to their respective purchasers which should not in general present any problems. Apart from this the amount of statistical work required in compiling the tables is the same as in Variant A.

3.101. Some difficulties may arise in deciding whether commodities should be classed as competitive or complementary. With a finer disaggregation of commodities more complementary imports are likely to appear. For instance, if the classification went no finer than a group, food, beverages and tobacco, this would be a competitive commodity; however, if the classification treated each of these three products as separate commodities in many economies the latter two would be classed as complementary. A further problem arises where there is a small amount of domestic production of a commodity which is imported in large quantities. In the U.K. example domestic production of crude oil was valued at £1m and imports at £375m. In such a case it seems appropriate to ignore the domestic production and treat this as a complementary commodity. The decisions on classification may thus, to a certain extent, be arbitrary in particular cases and may need revision over time if domestic production ceases or commences. Finally, it should be noted that the classifications will vary between economies and this may cause some slight problems in international comparisons.

5. <u>Variant D: Separate tables for imported and domestic products</u>

3.102. If it is possible to classify all imported goods by commodity as in variants A and C and also by purchaser as in variant B, two entries can then be recorded for each transaction - one for domestic products and one for imports. This is illustrated in Table 3.12 and in disaggregated form in Table 3.13. The total intermediate demand of 236 shown in Table 3.9 is here shown as consisting of purchases of 199 domestic products and 37 imported goods, and similarly for other sectors of demand. The imports entering each sector which were shown in the last row of Table 3.10 are transferred to the commodity row where they appear as a matrix rather than as a row. There are thus two input-output tables - one relating to inputs of domestic products and one to inputs of imported products. Similarly, there will be twin vectors for each final demand category.

3.103. The statistical requirements of this approach are demanding but the results make possible a considerable flexibility in the treatment of imports and permit a very clear analysis to be made of the impact of demand on home and foreign supplies. The records of firms and other purchasers generally do not contain information as to the origin (domestic or foreign) of a purchased commodity; indeed, a firm may not even know the origin if the commodity has been purchased from a wholesale dealer. The necessary data could perhaps be obtained by sample enquiry or a very detailed study of inputs combined with inferences by official statisticians as to their origin. It is important in constructing such tables not to simply assume the same domestic/import ratio for each transaction; to do

Table 3.13. Simplified input-output table and related accounts showing domestic and import flows, U.K. 1963 (£ hundred million)

			Commodities							Commod. tax	Industries							Consumption		Capital formation		World	Totals
			1	2	3	4	5	6	Total		1	2	3	4	5	6	Total	Pers.	Govt.	Stocks	Fixed		
Commodities	1. Agriculture, fishing and food	d									14.3	0.3		0.4		0.4	15	31	1	1		3	51
		i									6.7	0.9		1.9			9	10		0			19
	2. Chemicals	d									2.5	5.2	4.0	3.1	1.5	2.9	19	4	2	-0	0	6	31
		i									0.8	2.2	0.3	0.6	0.2	0.2	5	0	0				5
	3. Metals & machinery	d									1.6	1.2	40.4	1.9	5.4	5.3	56	7	9	1	18	23	114
		i									0.0	0.0	5.5		0.3	0.2	6	1	0	-0	2		9
	4. Other manufactures	d									2.0	1.3	3.8	17.0	7.2	6.6	38	19	2	0	0	7	66
		i									0.0	0.1		5.5	0.8	0.2	7	2		0	0		9
	5. Mining, construction and utilities	d									1.1	2.7	2.8	2.1	11.9	3.7	24	12	4	0	25	1	66
		i									0.1	3.9	1.2	0.2	0.0		5	0		-0			5
	6. Transport & services	d									5.9	3.6	10.3	6.9	4.4	16.3	47	85	4	0	3	16	155
		i									0.2	0.2	1.5	0.5	0.1	2.4	5	0					5
	Total	d									27	14	61	31	31	55	199	158	22	2	46	56	483
		i									8	7	9	9	1	3	37	13	0	0	2		52
Commodity taxes											-2	0	1	1	1	5	6	26	1	0	1	-0	34
Industries	1. Agriculture, fishing and food		51	0			0	1	53														53
	2. Chemicals		1	30	0	0	1	0	31														31
	3. Metals & machinery			0	114	0	0	0	116														116
	4. Other manufactures			0	0	65	0	0	66														66
	5. Mining, construction and utilities			0	0	0	65	1	66														66
	6. Transport & services						0	151	151														151
	Total		51	31	114	66	66	155	483														483
Indirect taxes			0	0	1	0	0	0	1	34													
Factor incomes											20	10	45	25	33	104	237	4	27		123	4	
Rest of the world			19	5	8	9	5	5	51		0	0	0	0	0	4	4	3	2				
Totals		d	51	31	114	66	66	155	483	34	53	31	116	66	66	151	483						
		i	19	5	9	9	5	5	52														

Notes: (i) d = domestically produced; i = imported.
(ii) These accounts appear with no distinction between domestic and import flows in Table 1.3.
(iii) This set of accounts is based on the 70 sector tables in "Input-Output Tables for the U.K. 1963" prepared by the U.K. Central Statistical Office,

this would bring no improvement in analysis compared with variant A. These problems of data can be overcome even in economies with a relatively high import content and the E.E.C. tables for 1959 and 1965 and the U.K. tables for 1963 all show twin entries throughout. The latter appear in aggregated form in Table 3.12 and in disaggregated form in Table 3.13.

6. The choice of variant

3.104. The decision as to which variant should be adopted in a particular case may well, as in other problems, involve a compromise between the statistical requirements of a particular method and the accuracy of the results obtained in application of the tables.

3.105. Variant A is the simplest to estimate as it does not distinguish purchasers by origin. It is unable, however, to provide accurate estimates of the import requirements of given vectors of final demand since it must assume the average import content applies to all buyers. Variant D, on the other hand, requires the maximum information but permits a very clear and accurate analysis of import requirements. Variant B requires relatively little less information than variant D, but is much less useful in application. Variant C provides a useful compromise as with little extra effort compared with variant A, it may be possible to identify a fairly large proportion of imports and treat them separately from domestic commodities. The problems of variant A will remain here as regards competitive imports and this approach should be regarded as a compromise until statistical resources permit the use of variant D. When variant D is being used in projection work it is necessary not only to project the input coefficients of the input-output table but also to separate these technical coefficients into domestic and imported components. This requires a separate study of import propensities and is considered further in Chapter V.

J. Exports

3.106. There are two factors to be considered in the treatment of exports in input-output accounts - valuation and re-exports. The details of exports in a commodity classification which are required for the export vector of final demand will normally be taken from international trade statistics. The valuation of these commodities will be f.o.b. at the border or the port of embarkation and in some cases this valuation will be the same as producers' prices. Often, however, this valuation will include distributive and transport margins involved in transferring the goods from the place of production to the point of departure from the country; this will particularly be the case where the exporter is not the producer. Such margins should be deducted from the value of the commodities as shown in the international trade statistics and entered as exports of distributive and transport services. Export commodities will then, like goods sold to domestic purchasers, be valued at producers' prices.

3.107. In some countries a small proportion of the goods entering the country are not for domestic usage but will subsequently be re-exported without being processed. Such goods should be omitted from the export vector and should also be deducted from imports. If, however, their value when exported is higher than when they were imported due to the addition of any handling margin, this margin should be recorded as an export.

3.108. An alternative treatment is to include such re-exports in both commodity imports and exports and to record a transaction in the lower (import) part of the rest of the world cell in the commodity row in Table 3.12. The disadvantage of this approach is that a transaction in which the country is not directly involved is recorded in its accounts. It is better to treat re-exports in the same way as foreign-owned goods transported by the country's shipping or airlines which do not enter the country and to omit them from the accounts.

K. Frequency of construction

3.109. The answer to the question as to how frequently input-output tables should be constructed is not a straightforward one. The ideal is the construction of annual input-output tables completely integrated with the estimation of national accounts as in the Netherlands and Norway. This, as was suggested in Chapter I will improve the accuracy of estimates of the whole of the national accounting framework including the input-output table. It will improve the consistency of statistical data and is the ideal framework. However, it must be admitted that this requires some statistical data which many countries do not at present collect on an annual basis and also requires considerable resources.

3.110. Viewed purely in the context of their use in input-output models, the estimation of full annual input-output tables is not essential. Although many applications of input-output tables require stable coefficients, most coefficients will usually be found to be sufficiently stable over the period of a few years to remove the need for full annual tables (see Section M of this chapter). It must be borne in mind in this context that in many countries it takes 2-3 years or longer to prepare input-output tables. One must consider carefully therefore whether one is justified in using resources to produce annual tables which will be at least 2-3 years out of date when first available and will contain coefficients very similar to those for adjacent years.

3.111. The next section discusses techniques which can be uased to update existing input-output tables from a past year to a current year, given a certain amount of national accounting data. These techniques can allow for regular changes in input coefficients (and, in some situations, for irregular changes) and have been shown to be reasonably accurate over a period of 5-10 years.

3.112. The actual frequency with which countries will find it desirable to estimate full input-output tables (as opposed to updating old tables) will depend on the resources available, the stability of input coefficients and the accuracy in their own particular case of updating techniques. In general, it should be found desirable to aim to produce an input-output table on full data every 5-10 years and to use updating techniques to produce annual tables with less detailed data.

L. Updating techniques

3.113. The complete estimation and balancing of an input-output table occupies a considerable amount of time and resources. It would be very desirable if it were possible to produce an accurate input-output table integrated with the national accounts which involved little more statistical information than is normally available each year in most countries. In this section we shall discuss methods which can be used to adjust an existing input-output table relating to a past year so that it fits with the rest of the accounting data of the current year. The basic method which was developed in Cambridge, U.K., about 1960,[5/] has become known as the R.A.S. method and goes a long way to removing the need for the compilation of annual input-output tables based on full data.

3.114. If we refer back to Table 1.2 we note that the basic input-output table, the absorption matrix (matrix 1) is at the intersection of the commodity and industry accounts, and thus if we have available disaggregated data for all the other entries in these two accounts, we shall be able to derive data on the total industrial demand for each commodity and the total intermediate purchases by each industry. National accounting data will be able to provide estimates of final demand vectors 2 through 5 and of primary inputs into industries, vector 16. International trade statistics will be a convenient source for vectors 6, 20 and 21, and production statistics should provide details for matrix 13. (It may be that normal production statistics provide details either of industry or of commodity outputs but not sufficient detail of secondary production to complete the make matrix. If this is the case it will be necessary to estimate the amounts of secondary production by reference to the last available make matrix. Any resulting errors are likely to be insignificant.) When information on commodity taxes is included all the entries in the commodity and industry accounts will be completed in disaggregated form except the input-output table. By deducting the totals of final demand

[5/] Department of Applied Economics, Cambridge University, Input-Output Tables Relationships, 1954-1966, Volume 3, in A Programme for Growth (Chapman and Hall, 1963). The description here is based on Chapter 3 of the volume.

(vectors 2 through 6) from the total supplies of commodities (matrix 13 plus vectors 14 and 20), the total intermediate demand for commodities - the row totals of matrix 1 - will be found. Again, by subtracting vectors 7, 16 and 21 from total industrial outputs given by matrix 13, the totals of commodity inputs into each industry - the column totals of matrix 1 - will be found.

3.115. We shall assume that there exists an input-output table estimated from full data for a past year and that the above information, the row and column constraints, has been collected for the present year and examine how this limited amount of data can be used to estimate a current input-output table.

3.116. This problem can be regarded as a statistical problem of adjusting a matrix to fit new constraints, and was first tackled by statisticians working as early as 1940. The basis of the R.A.S. method suggested in an input-output context by Stone consists of finding a set of multipliers to adjust the rows of the existing matrix and a set of multipliers to adjust the columns so that the cells in the adjusted matrix will sum to the required row and column totals relating to the later current year. It is assumed in the initial presentation that each element, a_{ij}, of the input-output coefficient matrix, A, is subject to two effects: (a) the effect of substitution, measured by the extent to which commodity i has been replaced by, or used as a substitute for, other commodities in industrial production, and (b) the effect of fabrication, measuring the extent to which commodity j has come to absorb a greater or smaller ratio of intermediate to total inputs in its production. It is further assumed that each effect works uniformly, e.g. that commodity i is increasing or decreasing as an input into all industries, at the same rate, and that any change in the ratio of intermediate to total inputs into a commodity has the same effect on all commodities used as inputs. The substitution multipliers which operate along the rows are denoted as vector r and the fabrication multipliers operating on the columns as vector s. Each cell in the base matrix, A_o, will be subject to these two effects and the new matrix A can thus be written as:

$$A_1 = \hat{r} A_o \hat{S} \tag{3.17}$$

where $\hat{r}$ and $\hat{s}$ are matrices with the vectors r and s in the diagonals.

3.117. In order to find the vectors r and s we must introduce the input-output flow matrix which we wish to estimate and its known row and column totals which we shall call u_1 and v_1. If we write X_1 for the flow matrix we have

$$X_1 = A_1 \hat{q}_1$$

$$= (\hat{r} A_o \hat{s}) \hat{q}_1 \tag{3.18}$$

The row and column totals of this matrix will be

$$u_1 = X_1 i$$

$$= \hat{r} (A_o \hat{q}_1) s \tag{3.19}$$

and

$$v_1 = X_1' i$$

$$v_1' = i' X_1$$

$$= r' (A_o \hat{q}_1) \hat{s} \tag{3.20}$$

3.118. These two equations (3.19) and (3.20) contain all the information available - the base coefficient matrix, A_o, the new row and column constraints, u_1 and v_1 and the current output levels, q_1. If these equations are solved simultaneously the values of the r and s vectors will be found and thus one calculates A_1 ($= \hat{r} A_o \hat{s}$) and X_1.

3.119. The solution to these equations which is most usually and conveniently adopted is an iterative one. The estimation process of obtaining X_1 from X_o thus in effect amounts to nothing more than a proportional adjustment of the base matrix successively along its rows and its columns until convergence is reached.

3.120. How the R.A.S. method functions will now be illustrated and later its properties will be outlined and some refinements discussed.

3.121. The simplified accounting matrix for the base year, 0, is given in Table 3.14(a) and all known information for the current year, 1, is given in Table 3.14(b). From (a) we form the coefficient matrix A_o in (c) and if initially we assumed no technical change our matrix for year 1 would be found by applying these coefficients to the new output levels and thus we have $A_o \hat{q}_1$ at (d). The row and column sums of this matrix, u and v, are calculated and compared with the known row and column sums for year 1, u_1^* and v_1^*. It can be seen that the assumption of unchanged coefficients would not be justified and we must now adjust the matrix at (d) to fit the new constraints - and thus find the r and s multipliers.

3.122. The first step is to calculate this (first) set of row multipliers, r_1 at (d) and to multiply the values in each of the rows so that the row totals meet the constraints as at (e). The column totals of the matrix at (e) are compared with v^* and a set of column multipliers are found which bring the column sums of the matrix to equal v^* at (f). It will now be found that, as a result of the operation of the column multipliers, the row totals of this matrix are no longer equal to u^*. A second set of multipliers, r_2, must now be applied to the rows so that they sum to u^*. The column sums will then require further adjustment and this process will continue until no further adjustment is necessary. The resulting matrix in this case is shown at (g), together with the r and s multipliers. These are, in fact, the product of the multipliers used at each of the successive rounds of adjustment. Thus the row multiplier for the first row, 0.884, is equal to $r_1 \times r_2 \times r_3 \times \ldots \times r_n$, in this case $0.873 \times 1.010 \times 1.002 \times 1.001 = 0.884$.

3.123. Each cell in the base matrix at (d) can be multiplied by its row

Table 3.14. Illustration of R.A.S. method

Note: For simplicity, we make no distinction between commodities and industries in this example.

(a) Accounts for year 0

	Commodity A	Commodity B	Commodity C	Total	Final demand	Total output
Commodity A	50	100		150	50	200
Commodity B	30	50	20	100	200	300
Commodity C	20	50	30	100	100	200
Total	100	200	50	350	350	700
Primary inputs	100	100	150	350		
Total output	200	300	200	700		

(b) Available data for year 1

	Commodity A	Commodity B	Commodity C	Total u*	Final demand	Total output
Commodity A				160	40	200
Commodity B				150	250	400
Commodity C				120	180	300
Total v*	100	250	80	430	470	900
Primary inputs	100	150	220	470		
Total output	200	400	300	900		

(c) Input-output coefficients for year 0, Matrix A_o

	A	B	C
A	.250	.333	0
B	.150	.167	.100
C	.100	.167	.150

(d) Year 0 coefficients applied to year 1 outputs, Matrix $A_o \hat{q}_1$ and calculation of first row multipliers

	A	B	C	u_1	u^*	$r_1 = u^* \div u$
A	50	133.3	0	183.3	160	.873
B	30	66.7	30	126.7	150	1.184
C	20	66.7	45	131.7	120	.911
v_1	100	266.7	75			
v^*	100	250	80			

(e) Adjustment of matrix along rows and calculation of first required column multipliers

Multiply the matrix at (d) along its rows by r_1 and obtain:

	A	B	C	$u_2 = u^*$
A	43.6	116.4	0	160
B	35.5	79.0	35.5	150
C	18.2	60.8	40.1	120
v_1	97.3	256.2	76.5	
v^*	100	250	80	
$s_1 = v^*/v_1$	1.027	.976	1.046	

(f) Adjustment down columns and calculation of second required row multipliers

Multiply the matrix at (e) down its columns by s_1 and obtain:

	A	B	C	u_2	u^*	$r_2 = u^* \div u_2$
A	44.8	113.6	0	158.4	160	1.010
B	36.5	77.1	37.1	150.7	150	.996
C	18.7	59.3	42.9	120.9	120	.992
$v = v^*$	100	250	80			

(g) Adjustment successively to rows and columns continues until both $u \equiv u^*$ and $v \equiv v^*$ with the final matrix:

	A	B	C	Row multipliers
A	45.3	114.7	0	.884
B	36.2	76.6	37.2	1.177
C	18.5	58.7	42.8	.902
Column multipliers	1.025	.974	1.054	

(h) Short-cut method with no economic meaning attached to r and s.

Matrix X_o at (a) is adjusted by R.A.S. to fit u^* and v^* with same result as at (g) but with different values of r and s

	A	B	C	Row multipliers
A	45.3	114.7	0	1.119
B	36.2	76.6	37.2	1.488
C	18.5	58.7	42.8	1.140
Column multipliers	.811	1.025	1.201	

and its column multipliers to yield the value of cells in the final matrix at (g). Thus the input of A into B which is 133.3 in the base matrix must be multiplied by its row multiplier (0.884) and its column multiplier (0.974) to give 114.7, the value in the final matrix.

3.124. It can be seen in this example that commodity B with a row multiplier of 1.177 is replacing commodities A and C as an input into intermediate demands. Looking at the column multipliers we see that commodity B is subject to a downward fabrication effect, i.e. it is using fewer commodity inputs and thus more primary inputs in its production process, perhaps because the production process has become more complicated. On the other hand commodities A and C are using more commodities and fewer primary inputs per unit of output, perhaps because their inputs are being purchased in a form which requires less processing before being transformed into the finished product. This would be the case if a firm in the motor industry began to purchase the engines for its vehicles from another supplier, whereas previously it had purchased the various parts and made the engine itself.

3.125. An alternative to the method shown in Table 3.14(a)-(g) is to start with the base year flow matrix and simply adjust this matrix to the new controls. This omits stages (c) and (d). The resulting matrix in this case, shown in Table 3.14(h), is identical to that obtained in (g) but the row and column multipliers are different. Indeed, the same meaning cannot be attached to the multipliers which are here covering changes in output levels as well as the two effects operating on the coefficients. Here we have:

$$X_1 = \hat{r}\, X_o\, \hat{s}$$

i.e.

$$A_1\, \hat{q}_1 = \hat{r}\, A_o \hat{q}_o\, \hat{s}$$

$$A_1 = \hat{r}\, A_o \hat{q}_o \hat{q}_1^{-1}\, \hat{s} \qquad (3.21)$$

Equation (3.21) is the basic R.A.S. equation (3.17) with output levels included. If one is interested in the values of r and s as measures of substitution and fabrication effects, this method is not suitable. If there is no such interest, one can regard R.A.S. simply as mechanical methods of adjustment and the use of this simpler method is to be recommended.

3.126. The mathematical properties of the R.A.S. method have been explored by Bacharach[6/] who shows that the method will produce a unique solution, which does not depend on whether rows or columns are adjusted first. If a particular cell was zero in the base matrix it will remain zero in the final matrix and no negative entries will appear in any cells in the final matrix. Further, experience has shown that the method soon converges on the solution and thus is not too demanding on computing time.

3.127. It is important to know how accurate this method of up-dating input-output tables can be and various tests have been carried out to compare tables obt ined by up-dating by R.A.S. with tables subsequently estimated for that year from full statistical data. Paelinck and Waelbroeck[7/] compared an input-output table for 1959 estimated by R.A.S. from the 1953 table with the actual 1959 table and Schneider[8/] performed similar tests with U.S. tables over the period 1947-1958. These tests showed that a matrix estimated by R.A.S. from a past year gave a somewhat better estimate of the later year than did the matrix of the past year, i.e. $\hat{r} A_o \hat{s}$ was closer to A_1 than was A_o. The number of errors in individual cells greater than 1 per cent was reduced from 17 to 9 in the Belgian case and from 121 to 103 in the American case, with a larger table.

3.128. The Belgian tests revealed certain major sources of the failure of the R.A.S. bi-proportional adjustment process and showed that if certain coefficients, generally identifiable in advance, could be derived

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6/ Bacharach, M.O.L., Bi-proportional Matrices and Input-Output Change, Cambridge University Press, 1969.

7/ Paelinck, J. and Waelbroeck, J., "Etude empirique sur l'évolution de coefficients input-output", Economie Appliqueé, 1965.

8/ Schneider, J., An evaluation of two alternative methods for up-dating input-output tables. (B.A. Thesis, Harvard, 1965).

exogeneously and excluded from the adjustment process a much better estimate could be obtained of the whole matrix.

3.129. This modified R.A.S. method, which incorporates some firmly based information about some of the cells in the matrix being estimated, is illustrated in Table 3.15. Here we assume that information is available on the input of B into A and hence there is no need for this cell to be involved in the adjustment process. The value of this cell is subtracted from the required row and column constraints, u^* and v^*, and this cell is set at zero in the base matrix. This forms the starting point for the R.A.S. adjustment as at Table 3.15(c). The R.A.S. adjustment procedure is then carried out in the normal way and when a solution has been reached at (d) the zero for the input of B into A is replaced by the known value 40.

3.130. The incorporation of exogenous data into the simple R.A.S. method will tend to improve the accuracy with which other cells can be estimated in the same rows and columns as the known cells. In the Belgian tests, six of the cells which the simple R.A.S. method had estimated badly were treated as if they were known exogenously and the modified R.A.S. method was used. The number of cells with errors of more than 0.5% was reduced from 20 in the simple R.A.S. method to 8 in the modified method.

3.131. The simple R.A.S. method will fail to produce an accurate estimate of A, if the assumption that the row and column effects work uniformly along rows and columns is not justified. This will be the case if commodity groups are not homogeneous but consist of, say, one product which is replacing a commodity in another row and one product which is not replacing another or is being replaced by some other commodity. The net effect as recorded by R.A.S. may be, say, an upward substitution effect on the row. However, if the two products are used in different proportions by consuming industries the substitution effect will not be uniform. A similar problem arising from lack of homogeneity was ob.erved in the Belgian tests where it was found that the cells relating to fuel inputs were badly estimated by the simple R.A.S. method. This was because coal, for instance, is used as a raw material in energy-producing industries but a source of power in

Table 3.15. Illustration of modified R.A.S.

(a) Accounts for year 0

		Commodity A	Commodity B	Commodity C	Total	Final demand	Total output
Commodity	A	50	100		150	50	200
	B	30	50	20	100	200	300
	C	20	50	30	100	100	200
Total		100	200	50	350	350	700
Primary inputs		100	100	150	350		
Total output		200	300	200	700		

(b) Available data for year 1

It is known from available statistics that the input of B into A is 40. The table is otherwise the same as Table 3.14(b).

		Commodity A	Commodity B	Commodity C	Total	Final demand	Total output
Commodity	A				160	40	200
	B	40			150	250	400
	C				120	180	300
Total		100	250	80	430	470	900
Primary inputs		100	150	220	470		
Total output		200	400	300	900		

(c) Year 0 coefficients applied to year 1 outputs

The known input of B into A is excluded from the matrix and from u* and v* before the adjustment process begins.

	A	B	C	u_1	u*	$r_1 = u^* \div u_1$
A	50	133.3	0	183.3	160	.873
B		66.7	30	96.7	110	1.138
C	20	66.7	45	131.7	120	.911
v_1	70	266.7	75			
v*	60	250	80			

(d) Final matrix

The R.A.S. adjustment continues as in Table 3.14(d)-(g) and when a solution is obtained the exogenous cell value of 40 is entered in the result:

	A	B	C
A	42.7	117.3	0
B	40	73.7	36.3
C	17.3	59.0	43.7

all other industries; in the latter case much more so than in the former, coal was being replaced by other fuels. This markedly different substitution rate seriously affected the results of the R.A.S. updating estimate.

3.132. It was stated at the beginning of this section that one of the advantages of the R.A.S. method is that it permitted one to estimate an input-output table for a year for which one had no data on industrial inputs. Although the modified R.A.S. requires some data on inputs these requirements are nothing like so great as the normal procedure of compiling input-output tables which, in many countries, call for special enquiries into the details of industrial inputs and outputs. Some data on inputs may be available from statistics collected regularly. Any such information should be used in the updating exercise; in general, one would not expect that an updating exercise would call for the collection of extra statistical information.

3.133. It is difficult to generalise as to which cells, rows or columns are well recorded by normally available statistics. In many countries fuel industries are well documented and information on the cells relating to fuel inputs should be available. The same may be true of the major inputs (fertilisers and feeding-stuffs) into agriculture. In some cases it may be thought worthwhile to hold a small special enquiry to obtain information on a particular cell or cells to be incorporated into an updating exercise. Alternatively, it may be possible to combine indices of price and volume movements with the base year value of a cell and obtain a measure of the value in the current year. Where there is no domestic production of a particular commodity it may be possible to obtain an estimate of the input of that commodity from international trade statistics, e.g. the input of crude oil into the petroleum refining industry.

3.134. In this context it should be noted that exogenous data on certain cells may lead to a greater improvement in the accuracy of the up-dating methods than data on other cells. Tests on U.K. input-output tables have shown that substantially improved results can be obtained when relatively small amounts of additional information about pre-selected major coefficients were incorporated into the up-dating exercise.[9/] It seems useful, therefore, to perform some tests of up-dating exercises to determine which are the most important cells in particular national tables. When these have been identified efforts can then be made to find exogenous data on these coefficients which will improve the accuracy of the up-dating exercise.

3.135. It should be noted that when one is using the modified R.A.S. method no economic significance can be attached to the values of the row and column multipliers. This should not prevent the use of the modified R.A.S. method. It may be preferable to regard the R.A.S. method simply as a statistical tool which can be used to adjust two-way tables. The basic method can be applied to many tables other than input-output tables. In some of these applications some meaning may be attached to the values of the multipliers as in the simple input-output case but in other cases the values of particular multipliers may have no significance.

3.136. If any available data can be incorporated into the up-dating exercise, the modified R.A.S. method will yield more accurate results than the simple R.A.S. method although even this will be a more accurate estimate of the current year matrix than would be the base year matrix. In many countries it should be possible to extract from statistics which are regularly available a certain amount of information on inputs or on trends in input coefficients since the year of the previous full table. It should be possible to produce an input-output table by this method quite soon after the end of the year, e.g. at the same time as the full national accounts are produced.

3.137. Such a table will obviously not be so accurate as a table estimated on full data (although even these will suffer inevitably from errors of measurement, particularly of smaller inputs), but should be sufficiently accurate to remove the need for a full table each year, particularly if some exogenous data on coefficients can be incorporated.

[9/] Allen, R.I.G., "The effects on intermediate output predictions of incorporating coefficients with known values into the R.A.S. updating method"; in "Aspects of Input-Output Analysis" edited by I.G. Stewart (Edinburgh University Press).

This is certainly likely to be true if one considers the use of the input-output table in any analysis or model-building and if one considers the input-output table in its role of assisting consistency in the statistical framework there may still be some value in an up-dated table. For instance, if a very high row multiplier is found for a particular commodity which does not fit in with knowledge of changes in industrial techniques,this would suggest either that final demand has been under-estimated or total output over-estimated resulting in an over-estimate of intermediate demand. Similarly, an unusual column multiplier might lead one to suspect one's estimate of primary inputs or total output of the sector.

3.138. It follows from these remarks that the application of the R.A.S. method, either simple or modified, will only produce an accurate estimate of the input-output table if the control totals are accurate. This requires accurate estimates of industry and commodity outputs, primary inputs (net output) and final demand. Any errors in this area of national accounting data are likely to be transmitted into the input-output table whose row and column totals are derived as residuals from these other vectors.

3.139. It is appropriate to end this section by recalling the conclusions of the previous section on the frequency with which input-output tables should be constructed. In order to make optimum use of the limited statistical resources available, in many countries, it should be quite satisfactory to produce an input-output table on full data every 5-10 years and to produce an annual table integrated with national accounts using the R.A.S. up-dating techniques described here.

M. Stability of coefficients

3.140. The importance of the homogeneity and proportionality assumptions was stressed in Chapter II, Section D, where it was emphasised that input coefficients should not be affected by the classification of sectors or by the level of output. The assumption that inputs vary in direct proportion to the level of output does not, however, also require that input coefficients do not change between one year and the next. The proportionality assumption for the use of input-output tables requires that in the year to which they refer input coefficients are constant, but there are a number of reasons for expecting that coefficients will vary over time.

3.141. Knowledge of the reasons for, and the extent of, such changes in input coefficients is very important in both the application, as well as in the construction, of input-output tables. There is, in most economies, an inevitable time-lag of at least two to three years after the end of a year before the input-output table for that year can be prepared and published. If the table is to be used in current analysis it is important to know whether and to what extent the coefficients have changed since the year to which they relate. Any information on the nature of changes in coefficients can be of considerable help in updating tables by methods described in the previous section and also in making projections of tables for a future year in connection with a planning exercise.

3.142. It should be noted that when the movement of input coefficients over time is being considered, the coefficients are almost invariably measured in constant prices. Movements in coefficients can then be properly regarded as technical coefficients measuring changes in the quantity of any given input and changes in recorded coefficients will not be the result of price changes.

3.143. Three major reasons can be given for changes in coefficients: (i) Technological change, (ii) Changing relative prices, (iii) Imperfect data.

3.144. The speed and extent of technological change in modern economies is one of the main reasons why input coefficients change over time. The replacement of many natural products by synthetic materials, the substitution of oil and/or electricity for coal, and the development of new products and automated production processes are amongst the more important aspects of modern technical change which have had, and are having, a marked effect on input coefficients in many economies. Some studies have also noticed a tendency for the column totals of the input coefficient matrix to decline over time and thus for the primary input coefficient to rise. This could indicate increased efficiency in the usage of material inputs or a trend towards increased fabrication of given material inputs associated with a more vertically-integrated production process and/or more complex finished products.

3.145. Changes in input coefficients often occur because the relative prices of inputs have changed. In many cases it is by changing relative prices that technical change has its effect on input coefficients and many changes in relative prices are the result of technical change. Changes in the relative prices of capital and labour inputs will affect the prices of different products by differing amounts (see Chapter V, Section B) and such changes in the relative prices of product inputs can cause changes in coefficients.

3.146. Thirdly, it is almost inevitable in practice that the homogeneity and proportionality assumptions will not be met perfectly, and that, as a result, changes in coefficients will occur over a period of years. It is quite likely, no matter how carefully a statistical classification is chosen, that the product-mix of a sector will change over time with consequent effect on coefficients. Also, although the proportionality assumption may be met for the relatively small changes in output levels likely within one or two years of the year to which a table refers, it may well be that over longer periods, when larger output changes occur, that inputs are no longer a linear homogeneous function of outputs.

3.147. A number of studies have been made in recent years to assess the extent and impact of changes in coefficients. Such studies usually take one of two forms. The first approach is to calculate what output levels would have been required to meet the final demands of one year with the input-output technology of another year. The differences between the actual and calculated output levels indicate the extent to which technology has changed. In algebraic terms we calculate:

$$q_o^* = (I - A_1)^{-1} f_o \tag{3.22}$$

where the subscripts refer to time. The differences between $q_o = (I - A_o)^{-1} f_o$ and q_o^* are due to technical change and the differences between q_o^* and q_1 are due to changes in final demand. It is thus possible to separate the change in output in each sector into that part which is due to technical change and that part which is due to final demand change. Carter[10/] used this approach to study technical change in the U.S. economy and Vaccara's study[11/] of the 76-sector U.S. tables showed that between 1947 and 1961 the mean change in sector outputs which was due to technical change was 35% - or 2.2% per year.

3.148. The work of Sevaldsen[12/] in Norway and Tilanus[13/] in the Netherlands is typical of the second approach which can only be used when input-output coefficients have been observed for a series of years. Both these studies showed that when the annual values of each coefficient were compared with their mean values, there was a considerable dispersion about the mean values; however, when the values were compared with a time trend the dispersion was considerably reduced. This dispersion about a linear time trend was still quite large but Tilanus found that over half the coefficients of the time trends were statistically significant. The median coefficient for all the observed cells indicated a tendency for coefficients to change by about 2 per cent per year.

3.149. It emerges clearly from these studies that input coefficients are not stable over time. Change does take place but it takes place fairly gradually, and there are good reasons for expecting gradual rather than rapid change. If technical progress manifests itself as a new product it will be necessary to overcome consumer resistance and this may be a slow process. This will apply equally if the new product is bought, not by final consumers, but by other industries. The situation is then similar

10/ Carter, A.P., Structural Changes in the American Economy (Harvard, 1970).

11/ Vaccara, B.,"Changes over time in input-output coefficients for the United States" in Applications of Input-Output Analysis edited by A.P. Carter and A. Brody (North Holland, 1969).

12/ Sevaldsen, P.,"The stability of input-output coefficients" in Applications of Input-Output Analysis edited by A.P. Carter and A. Brody (North Holland, 1969).

13/ Tilanus, C.B., Input-Output Experiments (Rotterdam, 1966).

to that which arises when technical progress brings new processes of production. A new technique will not be adopted by all firms in an industry immediately: the advantages of the new technique may need to be considerable to justify replacing existing capital equipment. Studies of the diffusion of innovation by Mansfield[14/] and others show that new methods usually spread relatively slowly.

3.150. The evidence that coefficients change limits the number of years to which an input-output table can be regarded as accurate for current applications. On the other hand the fact that coefficient changes appear to be fairly gradual enables one to extend the useful life of any table. Further, the fact that changes are not only relatively slight but are also fairly smooth will increase the accuracy of up-dating techniques. When one considers some of the major technological changes which have taken place in the post-war period alongside the fairly gradual change in coefficients observed in this period it seems reasonable to conclude that in projecting a table 5-10 years ahead one need not be too concerned about the accuracy of the projection being reduced by a technological advance which has not, at present, been made. Most technical changes which are likely to make projected coefficients different from present coefficients will probably already be known. It is true that it may not be easy to assess accurately their impact, but it seems, in general, unlikely that a process, currently unknown, will have a major impact within the projection period.

N. Tables at constant prices

3.151. In the pure theory of input-output the coefficients relate to the physical quantities of commodities used in producing a given physical quantity of another commodity. Almost all tables are, however, prepared in money values. Commodities are too heterogeneous to permit a purely quantitative measure and some, such as inputs of services, are not measurable in quantitative terms. Any changes in prices between one year and another will alter the recorded value of transactions and it is therefore

[14/] Mansfield, E., Industrial Research and Technological Innovation (Norton, 1968; also Longmans).

necessary to remove such price effects and to construct tables at constant prices.

3.152. Two principal reasons can be distinguished for the importance of constant price tables. Firstly, although input-output tables are measured in money values it is important that the coefficients should be interpreted in physical terms. These coefficients are technical coefficients and the inputs should be dependent upon the level of the corresponding output in physical terms alone and the inputs should be invariant with respect to price changes. (This does not rule out the possibility that a change in the relative price of two inputs may result in a technical change which will enable the cheaper to be substituted for that which is now relatively more expensive.) In a planning model using input-output it may be desired to project observed trends in coefficients into the future in order to obtain an accurate prediction of the input-output table in a future year. The trends which are observed and projected must be changes in physical or technical coefficients and must not reflect any changes in the prices at which the physical inputs are valued. This can be achieved only if the input-output tables are valued in constant prices.

3.153. The second reason why it is important to revalue input-output tables at constant prices is so that they can form part of the whole statistical system of price and quantity index numbers. This use of input-output tables is discussed in Chapter 4 of the S.N.A. and is also outlined in Chapter V, Section G of this volume.

3.154. When an input-output table is being revalued in the prices of some other year - the base year - it is essential that this is done at the same time as the remainder of the production accounts are being revalued. The vectors of final demand and the supply side (i.e. industry and commodity outputs and imports) should all be revalued simultaneously with the input-output table. Only in this way can one be sure that the supplies and demands for all commodities will balance in constant as well as current prices. To attempt to revalue the input-output table alone might well, given the imperfect nature of many statistics, result in implied price indices for final demand which are inconsistent with other data. The consistency which is necessary for accuracy will only be achieved if the whole revaluation is carried out as a single exercise. Recommendations and priorities for the construction of national accounts at constant prices are given in Chapter 8 of the S.N.A.

3.155. This approach is particularly important in view of the lack of homogeneity of commodities in most industrial classifications. The product-mix of a given commodity group may well vary between purchases for intermediate demand and final purchasers and also between various industrial purchasers. Unless the prices of each of the products in the group change at the same rate the price change between one year and another will be different for each of the purchasers. It will, therefore, not be possible in many cases to use a uniform price index over all the entries in a particular row as would be the case with a homogeneous commodity. Indeed, if the availability of detailed statistics permit, as many cells as possible in the input-output table should be separately revalued.

3.156. It should be noted that although one must require the commodity accounts to balance both at current and constant prices, the same cannot be said of the industry accounts. Industrial outputs can be revalued to the prices of another year as can the inputs of commodities into industries but great conceptual problems arise over any attempt to revalue factor incomes in the prices of another year. If this cannot be done then complete revaluation of industrial inputs and a balancing of them against industrial outputs and cross-checking is not possible. Any attempts which are made to revalue factor incomes and, in particular, the profit surplus, are necessarily arbitrary and often conceptually weak. The standard solution to measuring factor incomes or value added at constant prices consists of subtracting the value of intermediate inputs at constant prices from the value of gross output at constant prices. Thus factor incomes are treated as the residual or balancing item in the industry accounts.

3.157. Tables and accounts at constant prices can be obtained either by applying quantity indicators to the base year data or by applying price indices to the current year data. In the ideal statistical system the results of these two approaches will be identical and in practice it is desirable to attempt both approaches wherever possible. There will be some commodities such as machinery or in the services sector where direct quantity measurements will be difficult and where only price indices can be used . However, in many cases, for total output, if not for sales to particular purchases, both approaches should be possible. Differences in the results will pinpoint weaknesses in the indicators being used and should thus lead to an improvement in the statistics.

IV. COMPILATION OF THE TABLES

4.1. In the first section of this chapter some general points relating to the compilation of input-output tables are discussed and in subsequent sections the principal data problems in each of the different branches of the economy are outlined. Reference should be made to the S.N.A. for a more detailed discussion of many of the points discussed in this chapter, particularly to the importance and methods of integrating the estimation of input-output tables with the rest of the national accounts. (See, for instance, paragraphs 8.36-8.38 and annexes 8.2 and 8.3.)

A. General

4.2. In compiling an input-output table the work must be planned well in advance. Since the table can incorporate many forms of economic statistics, in designing a statistical system the data requirements of an input-output table should be borne in mind. There are several important points which must be considered before the work of compilation is begun.

4.3. The ultimate users of the table should be consulted, since to a large extent the uses to which the table will be put determine its design and provide the criteria for the methodology of its compilation. In many countries, committees composed of the representatives of such organisations as the Ministries, the Planning Board, and Economic Research Institutes are set up to deal with the methodological questions.

4.4. The availability of the basic material must be considered; a comparison of the available data with the requirements of the table will indicate the additional information which must be obtained. Frequently, data may exist in published or unpublished form which can be recompiled or reclassified. Decisions must be made on the scope and methods of special surveys necessary to fill the statistical gaps. When this has been done a preliminary estimate can be made of the costs of compilation.

4.5. In a task requiring attention to such varied detail, there must be effective co-ordination of effort between the organisations and individuals responsible. A common pattern of division of labour is to assign responsibility for a sector or a number of related sectors to one group, or organisation. The central co-ordinating authority may be a census office, national accounts office, or planning office.

4.6. So far as the classification of sectors is concerned, the level of detail of the worksheets must be determined, even if it is expected that the table in its final form will be more aggregated. Frequently, a more detailed classification (at the level, for instance, of the industrial census classification) makes it easier and improves the quality of the results. Some other advantages of working on a detailed basis may be noted. Completed input-output tabulations will always be historical in nature. In their application to analytical problems an effort will ordinarily be made to take into account the technological and other changes occurring after the base period of the study. It is obviously easier to revise and maintain detailed tabulations. Furthermore, a highly detailed tabulation permits adaptations to particular needs.

4.7. Control totals should be determined for every sector in value terms and where possible in physical units. When a sector consists of a group of establishments, then the value of gross output (total shipments plus change in inventories), should be taken as the control total. (The data in physical units may help to establish the distribution pattern of the sector.) Since total output is defined to be equal to total input, then the establishment of a control total for output automatically provides a control total on the input side as well. In addition, intermediate input control totals which are often established are the total value or cost of materials and value added, which are usually available from industrial censuses. Much of the data on control totals will be available from the national accounts when these are being compiled in line with the S.N.A.

4.8. There are two general approaches to the task of obtaining a complete description of inter-industry flows in an input-output table. One approach is to begin by distributing the output of each sector to all other sectors, a second is to fill out the columns of the chart from cost data. In other words, one can analyse the sales pattern of an industry or the cost structure. It is even more satisfactory if both approaches can be adopted, so that two independent estimates are made for each cell. This makes necessary what is perhaps the most difficult part of the compilation of the table - the work of detailed reconciliation. Given sufficient information, a third approach is to fill in simultaneously row and column data for particular groups of commodities. If only one approach is used, the work of reconciliation becomes much easier. Usually, the cost structure is the only existing source of information. The lack of sufficiently detailed information on the distribution of output to consuming industries in most countries is one of the major problems in completing the tabulation of inter-industry flows. The statistical reconciliation of all the rows and columns is a delicate task: a change in any one cell may lead to changes throughout the entire table. Accordingly, it should be carried out by some highly qualified officers of the unit responsible for the final drawing up of the table.

4.9. At the time of planning the entire study, it is worthwhile drawing up a list of the worksheets and tables which will be used. For example, the following set of tables may be prepared:

A. <u>Worksheet Tables</u>

Absorption matrix
Make matrix
Import table
Distribution margins table
Transport margins table

B. <u>Tables for Publication</u>

Absorption matrix
Make matrix
Import table
Distribution margins table
Transport margins table
Row percentage distribution table

Direct input coefficient matrix

Inverse coefficient matrix

Naturally, the tables for publication may be slightly more aggregated than the corresponding worksheets.

B. Manufacturing, mining and energy

4.10. The manufacturing sectors of input-output tables prepared in most market economy countries are based upon establishment data (the only exception being Japan, which uses the commodity approach) while most of the centrally planned economies use the enterprise as the basic statistical unit. So far as homogeneity is concerned, the enterprise unit in the centrally planned economies is not very different from the establishment unit in market economies.

4.11. If the national accounts are being compiled in line with the S.N.A. much of the required data on industrial outputs and inputs will be essentially the same. In any case the most usual procedure is to depend heavily on industrial inquiries. These are extremely detailed annual inquiries in some countries which supply the basic information for the manufacturing, mining and energy sectors.

4.12. According to international recommendation,[15/] the census should supply the following data:

(a) Gross output, which together with the import of similar commodities gives the control totals for the sector rows and columns.

(b) Value of output of each of the principal products.

(c) Cost of materials consumed and payments for services rendered during the period.

(d) Value added.

(e) Value of the principal fuels and raw materials used.

(f) Wages and salaries and other compensation paid to employees.

4.13. From this information, the following control totals and sub-totals can be established:

15/ Department of Economic and Social Affairs, United Nations, Recommendations for Industrial Statistics, (Series M, No. 48, New York, 1968).

Input	Output
Cost of materials etc. consumed	Gross value of output
Individual materials	Primary products
Fuels	Secondary products
Purchased electrical energy	Miscellaneous receipts
Expenditures on contract work	Contract work
Census value added	Repair work
	Sale of scrap, etc.
	Non-industrial activities
Total input	Total output

4.14. The classification of the individual materials should be detailed enough so that materials produced in different industries can be distinguished, and yet it should be sufficiently broad so that there is not a large residual "Miscellaneous" category of inputs. The classification outlined in the S.N.A. in Table 5.2 and paragraphs 5.82-5.85 and 6.49-6.52 should be used as a guide.

4.15. It should be noted that, where data are collected on the basis of shipments and purchases rather than production and consumption, the necessary adjustment should be made to take account of possible inventory changes.

4.16. When the basic data sources are inputs it is often impossible to separate imported from domestically-produced elements. This is particularly true in the case of competitive imports, although it is less true of complementary imports. Usually only the total supply of a competitive import is known (from customs sources), although often the nature of the good (e.g. a consumer good or investment good) permits the identification of its destination.

4.17. To trace the distribution of the output of one sector to others requires a knowledge of its intermediate and final uses. Estimates of final uses can be obtained from statistical surveys of retail sales, household expenditures, new fixed assets, foreign trade, etc. The feasibility of tracing the intermediate uses depends upon the organisation of the different channels through which goods reach their final consumers. In the United States a sample inquiry was made to obtain information on the disposition of the output of the principal products of each industry. Data were classified according to the following eleven types of consumers: company-owned sales branches, sales offices, and administrative offices; company-owned merchant wholesale establishments; all other wholesalers; other manufacturing plants of the same company (inter-plant transfers); all other manufacturers; company-owned retail stores; all other retailers; individuals (household users) and farmers; industrial, construction, institutional, and commercial users; state and local and federal governments; exports. In Hungary, statistics of the distribution of the output of industrial enterprises were expressly designed to satisfy the requirements for the preparation of the input-output table. Industrial units record their sales according to the sector to which the purchasing enterprise belongs. To facilitate this procedure, the Statistical Office provides the producing enterprises with a classification scheme, showing the sector to which each enterprise belongs.

C. Agriculture

4.18. The procedure for estimating input and output totals in agriculture are similar to those used in estimating the contribution of agriculture to the national income. In most countries, the contribution of agriculture to the national income is estimated by the "product method", according to which the gross value of output for each product is estimated, and value added is obtained after the deduction of the cost of all non-factor inputs from the sum of all output values. These three national accounting measures, gross value of production, total cost of non-factor inputs, and value added can be used directly as control totals for the input-output table. The details of the non-factor inputs into agricultural sectors tend to be rather simple compared to that of manufacturing sectors; the principal materials used consist of seeds, fertilizers, feeding stuffs, pesticides and fuel for agricultural machinery.

4.19. The total consumption of seeds can be estimated from sample surveys of areas sown under each crop to get information about the consumption of seeds per unit of crop area. Consumption of feeding stuffs can be estimated from the number of head of livestock, and that of fuel from the stock of agricultural machinery on farms. Consumption of fertilizers and pesticides may be estimated from domestic production and net imports. The 1970 World Census of Agriculture proposed that the following items should be enumerated: crops (area and production), livestock and poultry (numbers), agricultural employment, power, machinery, and transport facilities.

4.20. The estimation of the distribution of output of agricultural products is more difficult. In many countries, statistics showing the distribution of agricultural products are lacking, so that it is frequently a matter of balancing the data estimated from the input side. According to the definition in the S.N.A. (paragraphs 6.19-6.21, 6.103) agricultural products consumed on farms and changes in farm inventories should be included in the value of total output while the growth of standing crops should be excluded.

D. Construction

4.21. Construction is one of the industries which in most countries is measured on an activity basis. The reasons for this are twofold: first, because there is a considerable volume of maintenance construction work carried out by non-construction establishments on their own accounts, and secondly because measurement of construction activity is usually based on the output of the activity (number of houses, etc.) rather than on the enumeration of some organisation, such as establishment or enterprise.

4.22. One possible source of input and output data is a census of the construction industry. In this case, the same recommendations as were made for industrial censuses in Section B of this chapter apply. The gross value of production should exclude the value of sub-contracted work and thus eliminate any double-counting. The distinction between new construction work done and repair and maintenance work is extremely valuable in allocating output, since all new construction output is considered to be final - a delivery to gross fixed capital formation. In the absence of direct information, such as a sample survey, repair and maintenance work can be allocated to each intermediate sector according to the distribution of buildings in each sector. Like the censuses of other industries, the census of construction should enumerate the most important materials consumed.

4.23. Where there is no census of construction, data for the construction sector in an input-output table must be estimated indirectly. One approach is to use the number of licences or permits which are usually issued to new private buildings; another estimate of building activity can be obtained from the consumption of building materials. Information on public works can be derived from the accounts of national and local governments. In the absence of a construction census, the input structure of the industry may be estimated by the commodity flow method. The input structure may be compiled in yet another way; knowing the composition of construction activity (i.e. number of dwellings, offices, factories, hospitals, highways, etc.) and then estimating from a sample survey the input requirements of each type of building, the total volume of inputs required for construction as a whole may be approximated.

E. Transport and related services

4.24. The total value of output of transport services should be defined to be equal to the sum of charges paid (received by carrier as actual revenues or as imputations) by the users of transport services. The total operating revenues of transport enterprises (rail, road, air, and water) are usually available from the transport statistics. These figures will provide the control totals but not the necessary sector breakdown.

4.25. The problem of allocating transport costs is similar in many respects to the problem of allocating trade margins among the various sectors. As noted in Chapter III, Section F, a complete matrix is desirable showing in each cell the transport costs incurred in the flow from sector i to sector j. Each column total in such a table indicates the total transport costs involved in shipping all the various inputs to each sector. The total of each row is equal to the total transport costs involved in distributing the output of the corresponding sector to its various destinations.

4.26. It can be assumed that data can be provided showing the sectors of origin of the principal commodities, and that the purchases of the transportation service on the same commodites may be established. The difficulty arises in allocating the transport costs among the industries purchasing the commodities. The output of a given sector may embrace several different commodities which fall into separate freight classifications and travel at substantially different rates, while the product mix of the sector may not be known. Furthermore, significant differences may occur in the transport costs of the same commodity because of differences in destination, size of shipment, etc. Bearing in mind these problems it is usually necessary to collect additional information, perhaps by sample survey, from the transport, producing and purchasing enterprises.

4.27. Operating revenues from the transport of passengers can be divided between households and business expenditures of the various sectors. It is sometimes possible to estimate the individual items of business travel expenditures from the accounts of enterprises.

4.28. Warehousing and storage margins are usually estimated for those commodities which are principally marketed through channels involving storage in warehouses.

F. Trade

4.29. The value of output of the trade or distribution sector is universally taken to be the difference between the total values of goods and services bought and sold, not the gross value of sales. Thus the sector, which is sometimes sub-divided into wholesale and retail sectors, is shown as selling to other sectors the value of the trade margin on each transaction. To make available this service, the trade sector absorbs as inputs some materials (fuel and paper) as well as factor and non-factor services. Unfortunately, the accounting practices of establishments and enterprises and the officially published statistics in most market economies do not permit the direct collection of the inputs (materials costs) and outputs (margins) of the trade sector. International recommendations[16/] also follow this practice. In centrally planned economies, the trade margins and cost structure of the sector can usually be derived from the balance sheets of trading enterprises. But even this information is usually insufficiently detailed, since to complete the row of the trade sector in an input-output table in which transactions are valued at producers' prices, it is necessary to know the destination as well as the origin of all merchandise passing through the trade sector. In fact, the most desirable solution calls for the preparation of a complete table of trade margins (see Chapter III, Section F).

4.30. In the absence of direct information, it is possible to estimate the trade margin by commodity groups and to distribute it among the sectors according to the commodity distribution. Another method is the so-called "mark-up" method, which is based upon a knowledge of both the producer's value and the purchaser's value of any flow, as well as the excise tax and the transport costs involved. Subtracting the latter two items from the difference between the producer's and purchaser's values gives the trade margin as the residual.

4.31. According to international recommendations[17/] on the census of distribution (trade), sales should be enumerated by type of commodity and wages and salaries and purchases should also be recorded. Although these recommendations are of some value, countries wishing to compile input-output tables should bear in mind the further statistical requirements of input-output tables in designing their census of distribution.

G. Services

4.32. Services comprise an extraordinary heterogeneous group of activities, including education, health, research institutes, libraries, recreation services, personal services, etc. Generally, the output levels are measured by gross receipts. Control totals may often be provided by

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16/ International Recommendations in Statistics of Distribution, Statistical Papers, Series M, No. 26 (United Nations publication, Sales No. 58.XVII.4).

17/ Ibid.

general economic and business statistics, but very little information is available on the output distribution of services to intermediate sectors. However, most services sectors distribute their output entirely to final demand categories which are fairly easily identified. Those services which distribute their output to other intermediate sectors must be the object of special studies. Likewise, the input side of the services sectors usually requires detailed investigation, since in very few cases are the statistical data available for this purpose.

4.33. In some countries the advertising industry is defined on an activity basis, and the advertising revenues of newspapers etc. are transferred to the advertising industry. The total expenditure of non-profit education and hospital services is taken to be the value of the output of these sectors, including the depreciation of building and equipment.

V. INPUT-OUTPUT APPLICATIONS

A. Analysis of production

5.1. The basic role of input-output analysis as the link between final demand and industrial output levels was outlined in Chapter II. It was shown there how the inverse matrix $(I - A)^{-1}$ could be used to assess the effects on the productive system of a given level of final demand. In this section it will be useful to disaggregate both final demand and primary inputs in order to show how the impact of the demand from various sectors on the productive system can be analysed.

5.2. It will simplify the discussion if we use the same hypothetical set of accounts as in Table 2.2. These are shown below in Table 5.1 with final demand separated into its 3 main vectors and two categories of primary input shown. No distinction is made between industries and commodities.

Table 5.1. Hypothetical input-output table and accounts

	Intermediate demand				Final demand				Total output
	Sector A	B	C	Total	Consumption: Personal	Consumption: Govt.	Capital formation	Total	
Sector A		20	45	65	30	5	-	35	100
Sector B	30		30	60	90	10	40	140	200
Sector C		80		80	40	20	10	70	150
	30	100	75	205	160	35	50	245	450
Primary inputs: Wages	30	80	45	155					
Primary inputs: Profit	40	20	30	90					
Primary inputs: Total	70	100	75	245					
Total input	100	200	150	450					

5.3. From the input-output flow matrix the inverse matrix $(I - A)^{-1}$ was calculated and is repeated in Table 5.2 for convenience.

Table 5.2. Inverse matrix $(I - A)^{-1}$

		Sector A	Sector B	Sector C
Sector	A	1.077	.257	.375
	B	.351	1.171	.340
	C	.141	.468	1.136

5.4. It will be recalled that each column of this matrix shows the total requirements both direct and indirect, of 1 unit of output of the sector in that column. Thus, the 30 units of consumer demand in Table 5.1 for product A will require the production of 32.3 (= 30 x 1.077) of A, 10.5 (30 x 0.351) of B and 4.2 (30 x 0.141) of C. Similarly it is possible to calculate the total output requirements of the consumer demand for 90 B and 40 C and by summing one can find the total output requirements of the vector of consumer demand in Table 5.1. The 90 units of B will require 23.1 A and 40 C will require 15.0 A so that the total requirement for A for consumer demand is 71 (= 32.3 + 23.1 + 15.0).

5.5. This result is the same as the matrix multiplication $(I - A)^{-1}$ c, where c is the vector of consumer demand. This calculation is performed for the other two sectors of consumer demand and the results are shown in Table 5.3 along with the output requirements of all the components of final demand.

Table 5.3. Destination of output

		Personal consumption	Government consumption	Capital formation	Total gross output
Sector	A	71	15	14	100
	B	130	20	50	200
	C	92	28	30	150

5.6. This table can also be regarded as giving an analysis of the ultimate destination of the output of each of the sectors. Thus 71 units of A are produced to meet consumer demand; only 30 are sold directly to consumers, the remaining 41 being required as inputs into the production of B and C. There are no direct sales of A to capital formation but 14 units are sold indirectly, having been incorporated as inputs into B and C.

5.7. It will be noticed that the totals of each of the final demand columns in Table 5.3 exceed the level of expenditure by those sectors. This is a result of the double-counting of production both as input and when incorporated into the final output. This duplication is avoided if the results are expressed in terms of net output rather than gross output. The ratios of net to gross output, i.e. primary to total input, can be calculated from Table 5.1 as 0.7, 0.5, 0.5 and if the gross output of each of the sectors in Table 5.3 is multiplied by its respective ratio, the results can be expressed in terms of net output. These are shown in Table 5.4 where it can be seen that the row totals are now equal to the primary inputs (net outputs) of the sectors and the column totals are now equal to the expenditure by each of the final demand sectors.

Table 5.4. Allocation of net output

	Personal consumption	Government consumption	Capital formation	Total net output
A	49	11	10	70
Sector B	65	10	25	100
C	46	14	15	75
Total	160	35	50	2[illegible]5

5.8. This gives a clear picture of the relation between the final demand sectors and the producing sectors. The columns of this table show the extent to which activity is required in the various industries as a result of the given level and pattern of expenditure, and the rows show the extent to which each industry is ultimately dependent on the level of demand in each of the final demand sectors.

5.9. This analysis can be extended further if the vector of primary inputs is disaggregated. The coefficients of wages and profits inputs to each industry can be calculated from Table 5.1. The wage coefficients for the three industries are: 0.3, 0.4, 0.3 and if the gross output requirements of consumer demand in Table 5.3 are multiplied by these coefficients we obtain the value of the labour input required to meet consumer demand. If this calculation is performed for profits and for the other sectors, final demand can be analysed in terms of primary input as in Table 5.5.

Table 5.5. Final output in terms of primary input

	Personal consumption	Government consumption	Capital formation	Total
Wages	101	21	33	155
Profits	59	14	17	90
Total	160	35	50	245

5.10. This shows, for instance, that consumer demand of 160 generated a demand for labour input and wage incomes of 101 and a demand for capital input and profits of 59. Alternatively, this table can be viewed from the point of view of costs rather than incomes; labour costs accounted for 63% of consumers' expenditure, for 60% of government expenditure and 66% of capital formation.

5.11. In Table 5.6 the results of this type of analysis for the U.K. in 1963 are shown. Although the main U.K. tables distinguish domestic and imported products in each cell, for this analysis imports were allocated to purchasers (variant B in Chapter III, section H), and thus are treated as a primary input. This table includes taxes on expenditure and the category "sales by final buyers". The latter records sales of second-hand goods and payments to government authorities for various services.

5.12. Here one is able to examine the composition (or cost structure) in terms of primary inputs of each of the final demand sectors. One can

Table 5.6. Final output in terms of primary input, U.K. 1963

Percentage

	Consumer expenditure	Government current expenditure	Capital formation	Exports	Total final output
Income from employment	40.1	81.9	59.3	51.2	50.6
Gross profits	25.8	12.6	23.9	24.4	24.0
Imports	17.0	8.8	14.3	19.2	15.7
Taxes on expenditure	14.5	3.0	5.3	2.8	9.7
Sales by final buyers	1.6	-6.3	2.8	2.4	-
Totals	100.0	100.0	100.0	100.0	100.0

Source: U.K. Input-Output Tables, 1963 Table 4. (The last two columns of the published table have been re-worked to exclude re-exports.)

note that labour costs accounted for 40% of consumers' expenditure, over 80% of government expenditure, and about half of the value of exports. Capital costs, as measured by gross profits, accounted for about one-quarter of each of the categories except government expenditure.

5.13. Finally, it should be emphasised that the basic role of the input-output inverse matrix $(I - A)^{-1}$ is to provide estimates of the output level in each producing sector which is required to supply a given vector of final demand - both the direct and the indirect requirements. It is thus possible to assess the likely impact on the various producing sectors of a postulated change in some components of final demand such as an increase in public authorities' expenditure or a change in consumer demand following variations in taxation levels or credit restrictions. This section has shown how the analysis can be extended to provide more information than output levels.

B. Prices and costs

5.14. In Chapter II, Section C it was shown that the price of products was dependent on the prices of other products used as inputs and the costs of non-produced or primary inputs and thus that the prices of products could be traced to depend solely on the costs of primary inputs, both the direct and the indirect primary inputs. The basic price equations (2.10) and (2.11) are written as:

$$p = A'p + \bar{y}$$

$$p = (I - A')^{-1} \bar{y} \tag{5.1}$$

5.15. The vector of primary input coefficients, $\bar{y}$, can be split, as in the previous section, into two main components, wages ($\bar{w}$) and profits ($\bar{r}$) and the above equation becomes:

$$p = (I - A')^{-1} (\bar{w} + \bar{r}) \tag{5.2}$$

5.16. If we perform this calculation using the data of the previous section we have:

Table 5.7. Components of price

	$(I - A')^{-1}$			Direct input cost		Total input cost		Total Cost = Price
				Labour ($\bar{w}$)	Capital ($\bar{r}$)	Labour	Capital	
				(1)	(2)	(3)	(4)	(5)
A	1.077	.351	.141	.3	.4	.506	.494	1.000
B	.257	1.171	.468	.4	.1	.686	.314	1.000
C	.375	.340	1.136	.3	.2	.589	.411	1.000

$$\begin{array}{c} A \\ B \\ C \end{array} \begin{bmatrix} 1.077 & .351 & .141 \\ .257 & 1.171 & .468 \\ .375 & .340 & 1.136 \end{bmatrix} \left\{ \begin{bmatrix} .3 \\ .4 \\ .3 \end{bmatrix} + \begin{bmatrix} .4 \\ .1 \\ .2 \end{bmatrix} \right\} = \begin{bmatrix} .506 \\ .686 \\ .589 \end{bmatrix} + \begin{bmatrix} .494 \\ .314 \\ .411 \end{bmatrix} = \begin{bmatrix} 1.000 \\ 1.000 \\ 1.000 \end{bmatrix}$$

(Note: The first entry in column (3) is obtained by multiplying the entries in the first row of the matrix by the corresponding entries in column (1) and summing the three products; the second entry in column (3) uses the second row of the matrix and column (1) and so on.)

5.17. It will be immediately apparent that the price per unit of output in each of three sectors is given as 1.000. The reason for this is that we have not defined units of output in physical terms; the unit of output in this example is £1 worth of output as the transactions in our tables have all been valued in monetary terms. The result is that our calculations state that the producers' selling price of £1 worth of output is £1. Most input-output tables are prepared in value terms and calculations similar to equation (5.2) will yield a set of prices equal to unity. This does not, however, mean that such calculations are of no useful application.

5.18. Three main uses can be distinguished:- (a) the total inputs, both direct and indirect, of labour and capital into each sector can be found; (b) it is possible to estimate the changes in the prices of output of each sector following a change in the rates of payment to primary inputs; (c) this approach forms the basis of some of the index number calculations in Section G of this chapter. The first two of these applications will now be discussed.

5.19. The direct labour and capital costs per unit of output in industry A are respectively 0.3 and 0.4 (columns (1) and (2) of Table 5.7). When allowance is made for the labour and capital cost component of the products used as direct and indirect inputs in the production of A the total labour cost per unit of output is 0.506 and of capital 0.494. These are shown in columns (3) and (4) of Table 5.7 along with corresponding figures for the other sectors. A comparison of columns (3) and (4) in Table 5.7 will show the relative importance of capital and labour costs in the price of each of the three products. Since there are no imports and no taxation in this example, the prices of the products will depend simply on the costs of the primary inputs, labour and capital, as these are the only inputs exogenous to the productive sectors. The price of all product inputs into a sector is traced back and broken down into its labour and capital cost components by using the inverse matrix $(I - A')^{-1}$. In this way the cost of each product is shown to depend ultimately on the cost of primary inputs - the proportions of which vary between sector depending on the relative labour and capital intensity of their production and of industries supplying them with product inputs.

5.20. The second application mentioned above enables one to answer questions of the type "What will be the effect on the price of each product if the wage-rate in industry B increased by 10 per cent?" If we assume firstly that no increases in costs are absorbed by producers but are passed on to purchasers in higher product prices and secondly, that there is no substitution by purchasers between products as a result of changes in relative prices or by producers between factors as a result of changes in relative costs, we can then proceed in our example as follows. The direct wage cost for industry B will be increased by 10 per cent from 0.4 to 0.44. We then perform the calculations as in Table 5.7 and obtain the results shown in Table 5.8.

Table 5.8. Effect on prices of an increase in wage-rate in industry B

Direct input cost		Total input cost		Total Cost = Price
Labour ($\bar{w}$)	Capital ($\bar{r}$)	Labour	Capital	
(1)	(2)	(3)	(4)	(5)

$$\boxed{(I - A')^{-1}} \times \left\{ \begin{bmatrix} .3 \\ .44 \\ .3 \end{bmatrix} + \begin{bmatrix} .4 \\ .1 \\ .2 \end{bmatrix} \right\} = \begin{bmatrix} .520 \\ .733 \\ .604 \end{bmatrix} + \begin{bmatrix} .494 \\ .314 \\ .411 \end{bmatrix} = \begin{bmatrix} 1.014 \\ 1.047 \\ 1.015 \end{bmatrix}$$

5.21. A comparison of column (3) here with column (3) in Table 5.7 shows that the increase in wage rate by 10 per cent in industry B has increased wage costs in all three industries. The total capital cost is unchanged and column (5) shows the new prices of each of the products. The largest increase occurs in industry B but increases of 1.4 and 1.5 per cent will occur in industries A and C. The assumptions underlying these calculations may not hold rigidly in practice but nevertheless this type of analysis is useful in that it can indicate where pressure on costs leading to price increases is likely to be felt and if such price increases do not come about, this analysis will enable one to see to what extent substitution between products and/or factors has limited the extent of the price increase.

5.22. This type of analysis can be extended to show the effect on product prices of changes in the rate of indirect taxes on inputs. The example used in this chapter has omitted taxation but reference to Table 1.3 shows how taxes enter the cost structure of industries in the same way as the primary inputs, labour and capital. In this case it is then possible to write equation (5.2) as:

$$p = (I - A')^{-1} (\bar{w} + \bar{r} + \bar{t}) \tag{5.3}$$

where $\bar{t}$ is the vector of taxation on inputs per unit of output. By varying the values of these taxation coefficients to correspond to different taxation rates, the effect on prices of the products can be assessed. The U.K. Treasury have made use of this type of analysis to assess where the effects of proposed tax changes will be felt.

5.23. A further extension of this price analysis will show the effect of a change in the price of imports on the prices of products. This can be seen most clearly if imports are treated as in variant B in Chapter III, Section H.3. Table 3.10 there shows that the full input structure of industries consists of home produced commodities, taxes, primary inputs (factor incomes) and imported commodities. The price equation can thus be written:

$$p = (I - A')^{-1} (\bar{w} + \bar{r} + \bar{t} + \bar{m}) \tag{5.4}$$

where $\bar{m}$ is the vector of import coefficients recording imported inputs per unit of output. Here the prices of home-produced commodities are seen to depend on the costs of all those inputs which are exogenous to the productive sectors. If a different vector $\bar{m}$, representing a different set of import prices, is entered in (5.4) the new set of domestic product prices can be calculated in the same way in which in Table 5.8 a different vector of wage coefficients was used.

5.24. An interesting example of an assessment of the impact of a change in import prices is provided by Barker's study[16/] of the effect of the devaluation of sterling on the prices of U.K. industrial outputs. In the input-output framework which he used imports were treated as in variant A in Chapter III, Section H.2. In this case the basic price equation (5.1) must be re-written as:

$$p_h = A'p_n + \bar{y} \tag{5.5}$$

where p_h is the prices of home produced commodities and p_n is the prices of inputs. The latter will be weighted averages of the prices of home produced commodities and the prices of imports, p_m, and we have:

$$p_n = \hat{b}p_m + (I - \hat{b})\ p_h \tag{5.6}$$

where b is the proportions in which total supplies of commodities are met from imports and (I - b) is the proportion of home to total supplies. Combining (5.5) and (5.6) we obtain:

$$p_h = A'\hat{b}p_m + A'(I - \hat{b})p_h + \bar{y}$$

which, on rearranging, gives:

$$p_h = \left[I - A'(I - \hat{b})\right]^{-1} (A'\hat{b}p_m + \bar{y}) \tag{5.7}$$

5.25. This rather complicated equation is similar to equations (5.2) through (5.4) in that it relates the prices of domestic products, p_h, to the prices and costs of non-produced inputs, p_m and $\bar{y}$, using the input-

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16/ Barker, T.S., "Devaluation and the rise in U.K. prices". Bulletin of the Oxford Institute of Economics and Statistics, Vol.30, No.2, May, 1968.

output matrix. In Barker's study a vector of post-devaluation import prices was inserted and a new set of home prices was calculated. The vector of import shares, b, was also amended where it was felt that import shares would fall as a result of devaluation. The calculated price increases compared well with the actual increases in the year following devaluation.

C. Import requirements

5.26. The four methods of treating imports in an input-output framework were discussed in Chapter III, Section H and the relative merits of the four variants for estimating the import content of demand were outlined. In this section we shall extend our hypothetical accounts of the previous two sections in order to illustrate how the impact on imports of a given vector of final demand can be calculated, using variants A and D.

5.27. These calculations are illustrated in some detail since the determination of the level of imports is often a vital part of an input-output exercise, particularly in economies where the balance of payments imposes a constraint on their domestic economic policies. Calculations such as these are useful in short-term analysis since they enable one to know the impact on both domestic output levels and on imports of a given vector of final demand or a given change in final demand induced by some policy measure. Medium or long-term planning models may also use the methods described here to determine import requirements in the target year of the planning period. In this case there may be a considerable amount of preliminary work needed before the calculations described here could be carried out. Changes are likely to occur in the input coefficients as a result of a technical change and in the import content of supplies due to changes in relative home and foreign prices. Some models, therefore, will prefer to introduce prices explicitly in order to forecast the relative shares of imports and domestic production in total supplies.

5.28. A given vector of domestic final demand generates a demand for imports in two ways. Firstly, there is a direct demand for imported consumer or capital goods; secondly, there is an indirect demand for imported inputs from all industries involved, both directly and indirectly, in supplying that part of the domestic final demand which is not met by imports. In the case of the final demand vector of exports there is only the second category - the indirect demand for imports. Since the treatment of home final demand differs somewhat from that of exports we shall consider the impact on imports of an assumed final demand vector consisting of home final demand of 100 for product B and export demand of 100 for A.

5.29. The hypothetical set of accounts with imports treated as variant A in Chapter III, Section H.2 is shown in Table 5.9. These accounts are very similar to those in Table 5.1 but some alterations have been made in order to include imports and exports.

Table 5.9. Hypothetical accounts: Imports variant A

	Commodity			Industry			Home final demand	Exports	Totals
Commodity				0	20	25	25	20	110
				30	0	30	140	50	250
				0	80	0	80	40	200
Industry	100	0	0						100
	0	200	0						200
	0	0	150						150
Incomes				70	100	75			
Imports	10	50	50						
Totals	110	250	200	100	200	150			

5.30. A distinction is made between industries and commodities but as there is no secondary production this presents no problems and the commodity x commodity input-output table will be identical to the commodity x industry (see Chapter III, Section B). The commodity x commodity coefficient matrix, A, can thus be written as $A = X\hat{q}^{-1}$. The notation is fully defined in the Appendix I. An asterisk (*) is used to indicate projected or assumed levels of demand, supply, etc., and subscripts (d) and (m) are used to indicate that part of a demand vector which is supplied from domestic production and imports respectively.

5.31. The sequence of calculations is shown in Table 5.10. The entries in columns (1) and (2) are obtained from Table 5.9 and this enables us to calculate the import ratio of home demand (b) in column (3). The assumed vectors of final demand are shown in columns (5), (6) and (7). The direct import content of home final demand, h^*_m, in column (8) is found by multiplying columns (3) and (5).

5.32. The second row of this table shows the input-output matrix in coefficient form and by multiplying each of the rows by the domestic content ratios from column (4), we obtain a matrix of domestic input coefficients, A_d. The inverse of this matrix is then calculated and this shows the domestic output levels required in each sector to produce one unit of output in each of the sectors. From this matrix we can calculate the output levels required to meet the assumed final demand in column (7); these output levels are in column (10). By applying the coefficient matrix A to these output levels we obtain X^* a matrix of required input-output flows (11) from which we obtain the required intermediate demand vector, z^*, in column (12). We apply the average import ratios from column (3) to this and obtain column (13), the intermediate demand for imports. The total demand for imports is found by combining the intermediate and final demands for imports into column (15).

5.33. Two points should be noted about this method. Firstly, the matrix which was inverted was A_d and not A. Since the purpose of the Leontief inverse is to calculate the repercussions on the productive system of a given output or demand, imported inputs must be omitted since they have no repercussions on the productive sectors. Secondly, in calculating the import content of both intermediate demand and final demand it was necessary to assume the same import contents in both cases. This, as was pointed out in paragraph 3.94 is one of the main drawbacks of variant A.

Table 5.10. Impact on imports of a given vector of final demand - imports variant A

	Imports m	Total home demand $z+h$	Import content of home demand b	Domestic content of home demand d	Assumed final demand: Home h^*	Assumed final demand: Export x^*	Assumed final demand: Total f^*	Import content of final demand $h^*_m = bh^*$	Domestic content of final demand $f^*_d = dh^* + x^*$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A	10	90	.111	.889	0	100	100	0	100
B	50	200	.250	.750	100	0	200	25	75
C	50	160	.312	.688	0	0	0	0	0

	Input coefficient matrix A			Domestic input coefficients $A_d = \hat{d}A$			$(I - A_d)^{-1}$		
A		.1	.3		.089	.267	1.040	.176	.304
B	.3		.2	.225		.150	.244	1.084	.228
C		.4			.275		.067	.298	1.063

	Required output levels $q^* = (I-A_d)^{-1} f^*_d$	Required input-output matrix $X^* = Aq^*$			Total intermed. demand z^*	Import content of intermed. demand $z^*_m = bz^*$	Import content of final demand h^*_m	Total import content m^*
	(10)	(11)			(12)	(13)	(14)=(8)	(15)
A	117.2		10.6	8.7	19.3	2.1	0	2.1
B	105.7	35.2		5.8	41.0	10.3	25	35.3
C	29.1		42.3		42.3	13.2	0	13.2
								50.6

Since each commodity group may consist of a number of products with different individual import contents, if the product-mix varies, as is likely, between intermediate and final demand, the use of average import content ratios will lead to inaccurate results.

5.34. A much more accurate estimate of import requirements can be obtained if full details of imports are available as in variant D, described in Chapter III, Section H.5. The hypothetical set of accounts used in Table 5.9 are shown in Table 5.11 as variant D with complete separation of imports and domestic products throughout the accounts.

5.35. The calculation of import requirements for this system is shown in Table 5.12. The import content of home final demand is shown in column (3) and has been calculated from the first two columns which are obtained from the basic data in Table 5.11. The import content of the assumed home final demand is calculated in column (7) and the domestic content of final demand (home and exports) is shown in column (8). In the second row of this table the matrix of domestic input coefficients, A_d, is shown. This is obtained by expressing the domestic entries in the input-output table in Table 5.11 as coefficients. The Leontief inverse of this matrix is next calculated and used to find, in column (11), the total output requirements of the final demand vector. In the bottom row of the table the matrix of imported input coefficients, A_m, is calculated from the basic accounts (12) and used to find the inputs of imports required to meet the calculated level of outputs. The total intermediate demand for imports is found by summing along the rows of this matrix at (13) and appears in column (14). The direct final demand for imports is added to this to give the import requirements in column (16) of the assumed vector of final demand.

5.36. A comparison of columns (14)-(16) in Table 5.12 with columns (13)-(15) in Table 5.10 shows that the composition of the total import requirements differs noticeably between the two approaches.

5.37. In these examples the total import requirements are very similar but this will not always be the case and the more accurate results will be

Table 5.11. Hypothetical accounts, imports variant D

		Commodity			Industry			Home final demand	Exports	Totals
Commodity	d				0	14	45	21	20	110
	i				0	6	0	4	0	
	d				20	0	20	110	50	250
	i				10	0	10	30	0	
	d				0	60	0	50	40	200
	i				0	20	0	30	0	
Industry		100	0	0						100
		0	200	0						200
		0	0	150						150
Incomes					70	100	75			
Imports		10	50	50						
Totals		110	250	200	100	200	150			

Note: d = domestic product

i = imported product

Table 5.12. Impact on imports of a given vector of final demand: imports variant D

	Total	Home final demand		Assumed final demand			Import content of final demand	Domestic content of final demand
		Imported	Import content	Home	Export	Final		
	h	h_m	b	h^*	x^*	f^*	$h_m^* = bh^*$	$f_d^* = f^* - h_m^*$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
A	25	4	.160	0	100	100	0	100.0
B	140	30	.214	100	0	100	21.4	78.6
C	80	30	.375	0	0	0	0	0

	Domestic input coefficients $A_d = X_d \hat{q}^{-1}$			Inverse $(I - A_d)^{-1}$			Required output levels $q^* = (I - A_d)^{-1} f_a^*$
	(9)			(10)			(11)
A		.07	.03	1.036	.172	.332	117.2
B	.2		.13	.215	1.076	.204	106.3
C		.30		.065	.320	1.091	31.7

	Imported input coefficients $A_m = X_m \hat{q}^{-1}$			Required inputs of imports $X_m^* = A_m \hat{q}^*$			Internal demand for imports Z_m^*	Final demand for imports f_m^*	Total import content
	(12)			(13)			(14)	(15)=(7)	(16)
A		.03			3.2		3.2	0	3.2
B	.10		.07	11.7		2.2	13.9	21.4	35.3
C		.10			10.6	0	10.6	0	10.6
									49.1

obtained with variant D which can allow for different import contents in each sector of demand - both between final and intermediate demand and within intermediate demand.

5.38. The use of variants B and C should be briefly discussed. Variant C is a special case of variant D where only complementary imports are treated separately. Competitive imports are classed with domestic products and the calculations will thus be similar to variant A with requirements for complementary imports following the pattern of variant D. The use of variant B is also similar to variant D; in place of a matrix of imported inputs as in the last row of Table 5.12, variant B will have a vector of import coefficients and the result will be intermediate demand for imports classified by purchasing industry and not by imported commodity. The total import requirements can be obtained but not the commodity details; this is the main drawback of this variant.

D. Investment and capital

5.39. In the discussion in all previous sections of this volume all the transactions in the input-output tables have been transactions in goods and services required for current production. No consideration has been given to inter-industry transactions in capital goods. In this section the input-output framework will be extended to cover not only the demands one industry places on another for goods and services for further processing, but also to include demands for capital goods which are required in order to add to the stock of existing capital equipment.

5.40. The demand for capital goods has previously been shown as a vector of gross domestic fixed capital formation which shows the total demand from all industries for capital goods but does not show the details of the purchasing industries (e.g. Table 5.1). It is possible, however, to show this vector, v, as a matrix V, which will record the sales of capital goods by industries in the rows to the purchasing industries in the columns. In this case the basic accounting equation (2.5) can be written as:

$$q = Aq + v + c$$

or

$$q = Aq + Vi + c \tag{5.8}$$

where c is final consumption, i.e. that part of final demand which is not capital formation. Therefore, we have now two matrices of inter-industry transactions, one relating to transactions in goods used in current production and the other relating to transactions in capital goods.

5.41. If we wish to use equation (5.8) to calculate output levels of industries required for certain postulated levels of final consumption it is desirable to relate the investment matrix, V, to output in some way. Since the level of investment in an economy is generally accepted to be a function of changes in the level of output rather than a function of the actual level of output, a coefficient matrix for capital goods will be expressed in a rather different form from the current input-output coefficient matrix, A. Since the total size of the existing stock of capital equipment is directly related to the level of output we may define a matrix, K, the entries in which, k_{ij}, will show the stock of goods produced by industry i which industry j holds per unit of its (full capacity) output. A column of this matrix describes what may be called the capital structure of an industry showing the proportions in which buildings, machinery, vehicles, etc., as well as inventories of raw materials, spare parts and other supplies are held by the industry in its production process.

5.42. The total stock of capital assets held by an industry at any time will be determined by its capital structure as shown in the capital coefficient matrix K, and its level of output. An increase in this stock is net investment by the industry and thus we see that investment will depend on the capital coefficients and increases in the level of output. If we write S as the total stock of capital equipment and use subscripts to denote time, we have:

$$S_1 = K\hat{q}_1$$

$$S_2 = K\hat{q}_2$$

$$S_2 - S_1 = K(\hat{q}_2 - \hat{q}_1)$$

$$V = K\Delta\hat{q}$$

5.43. This provides the necessary link between the investment matrix, V, and output and if we combine equations (5.8) and (5.9) we have:

$$q = Aq + K\Delta q + c \tag{5.10}$$

where Δq is the increase in output between the year in question and the following year.

5.44. In order to solve this equation and use it in calculations it is necessary to eliminate the term Δq. We can write, with time subscripts,

$$\begin{aligned} q_1 &= Aq_1 + K\Delta q_1 + c_1 \\ q_2 &= Aq_2 + K\Delta q_2 + c_2 \end{aligned} \tag{5.11}$$

and by subtraction obtain:

$$\begin{aligned} \Delta q_1 &= q_2 - q_1 \\ &= A\Delta q_1 + K(\Delta q_2 - \Delta q_1) + \Delta c_1 \end{aligned} \tag{5.12}$$

If we assume the output grows on a linear path over time the increase in output levels from year 1 will be the same as from year 2, i.e. $\Delta q_1 = \Delta q_2$ and (5.12) will become:

$$\Delta q = A\Delta q + \Delta c$$

i.e. $$\Delta q = (I - A)^{-1}\Delta c \tag{5.13}$$

By inserting this value of Δq in (5.9) we see that the level of investment can be written as:

$$V = K(I - A)^{-1}\Delta c \tag{5.14}$$

5.45. By combining (5.10) and (5.13) or (5.8) and (5.14) we obtain:

$$q = Aq + K(I - A)^{-1}\Delta c + c$$

which solving for q gives:

$$q = (I - A)^{-1} [c + K(I - A)^{-1}\Delta c] \qquad (5.15)$$

5.46. In this equation the output levels are dependent on the inter-industry transactions involved in supplying both final consumption and the capital requirements of the future growth in consumption. The first term in equation (5.15) is $(I - A)^{-1}c$ and this shows the total output requirements (both direct and indirect) of the assumed level of final consumption. The demand for capital goods is not included here but appears in the other term of the equation. The future growth in consumption c will require output levels $(I - A)^{-1}\Delta c$. In order that this additional output may be produced in the following year, capital goods must be produced in the current year. This demand for capital goods is determined by the capital coefficient matrix K and the level of investment is given as in (5.14) as $K(I - A)^{-1}\Delta c$. The total output requirements of this level of investment demand is the second term in (5.15): $(I \quad A)^{-1} K(I - A)^{-1}\Delta c$.

5.47. A simple model based on equation (5.15) is able to estimate future output levels required for a certain assumed future level o consumption and growth in consumption. No assumptions need to be made about the level of investment demand; the use of the capital coefficient matrix, K, enables one to calculate investment demand within the equation.

5.48. The introduction of the capital structure into the input-output system described above is probably the simplest way in which this additional input matrix can be incorporated. There are various more cc lex treatments but consideration of these is beyond the scope of the present volume. Some qualifications should, however, be made to the above approach and these will indicate ways in which this approach may be developed.

5.49. Firstly, it should be emphasised that it is possible to estimate investment demand using a production function approach rather than capital matrices. In this case an initial estimate of demand will be made and combined with the assumed level of final consumption; the output levels required for this level of final demand will be calculated using equation (5.8). A production function (e.g. of the Cobb-Douglas type) can then be used to calculate the investment requirements of these output levels. This second estimate of investment demand can then replace the initial estimate and a second set of output levels can be calculated. This iterative procedure will usually soon converge to a solution to yield estimates of investment demand and output levels.

5.50. Secondly, the investment demand which we have been considering is that which arises due to a desire to increase the stock of capital. There is also usually a demand for capital goods to replace certain parts of the existing stock of capital whose useful working life is exhausted. These two categories of investment can usefully be termed extension and replacement demand. Given knowledge of the age structure of the existing stock of capital the level of replacement demand in a future year can be calculated outside the model equation (5.15) and can be entered as part of the exogenous assumed level of final consumption demand. Investment to extend the capital stock will then be estimated as outlined above. It should be noted that this investment will be negative for an industry whose future output is declining, although replacement investment would always be positive.

5.51. Thirdly, it was assumed that all capital goods required in one year would be produced during the previous year. However, for certain types of capital equipment the gestation period may well be longer than one year. For instance, the building of an electricity generating station may produce demands for building and construction activity three or four years before it is completed and demands for machinery during the following two years. The demands for capital goods generated through matrix K in (5.15) will thus have to spread over four years and the level of investment in the future year in question will thus be dependent on the rate of output growth and capital requirements over the following four years. Equation (5.15) can be adapted to incorporate these complications.

5.52. Fourthly, it may not be felt reasonable to assumed that the capital input coefficients do not change over time as was done in equation (5.9) where these matrices have no time subscript. Annual changes in the capital structure of an industry may not be large but if allowance is to be made for them this requires that investment demand in (5.9) must allow for changes in K in addition to the growth in output.

5.53. Finally, we should note that equation (5.15), although it introduces the link between growth in output and investment in a future year, does only provide estimates of output levels in one future year. The continuous linking of required output levels in a series of successive years via capital requirements can be achieved only with a fully dynamic input-output system. This will provide estimates of output levels for a series of years and thus show the path along which the economy can be expected to move from the current year to the future year at the end of the projection period. Dynamic input-output systems are discussed in Section F of this chapter.

E. Planning models

1. Introduction

5.55. One of the most important uses of input-output tables is in medium-term planning where the aim is to obtain a detailed forecast of supplies and demands within the economy for a "target year" five to ten years ahead. A certain growth rate in total national product and in the main components of demand is assumed over the planning period and an input-output model can then be used to disaggregate the forecast and obtain estimates of industry output levels as well as import and factor input requirements; the latter can be balanced respectively against expected exports and factors, especially labour, availabilities to test whether or not the assumed growth rate is feasible.

5.56. The exact benefits derived from such an exercise may vary according to the nature of the economy where it is being carried out. In a centrally-planned economy it enables the authorities to plan production accurately at the level of the industry and, by correctly phasing investment, to ensure optimum utilisation of capacity and resources. In a capitalist economy where the decisions on production and capacity are made by a large number of individual firms, this is not possible. However, a plan based on an input-output model will help to ensure that these decisions will all be made within the same framework of growth rates. In this way planned production in individual industries should neither fall short of, nor exceed, the demands for their products from final purchasers and other industries, i.e. consistency will be achieved between industries.

5.57. This establishment of consistent targets is one important aspect of "indicative planning" in free market economics. The other important aspect is the aim of increasing the growth rate of the economy. The target growth rate will then be set higher than the prevailing growth rate and it is hoped that the consistent framework of the plan will enable industries to plan their future activities more efficiently and with greater confidence, and further that the details of the plan may pin-point particular aspects of the economy which require special attention (e.g. shortage of labour of a particular type). In these ways it is hoped that the higher growth rate will be achieved.

5.58. It is probably correct to say that of the large number of planning models which exist in the industrialised economies of the world, none are identical. Although the basic aim - a disaggregated forecast of supplies and demand - is usually the same the details may vary considerably as will also the emphasis each country places on particular aspects of the planning exercise. In an economy like the U.K. with a large element of foreign trade, considerable emphasis will be placed on the import requirements and the likely balance of payments implications of a planned target rate of growth. Alternatively, emphasis may be placed on labour requirements compared with the expected working population not only in total but perhaps also in detail, e.g. by type of skill or geographical region. However, the general principles of an input-output plan and the relationships to be considered are common to all planning exercises and it is these which will be considered in this section.

5.59. Diagrams 5.1 and 5.2 are based on the model of the U.K. economy developed at the Department of Applied Economics, Cambridge.[17/] In other models many aspects may vary - the sequence of calculations; what is considered exogenous and endogenous; the equations which link variables - butthe model described here can usefully illustrate the general methodology of an input-output plan.

5.60. In order to simplify the description we shall first of all describe the model shown in Diagram 1 which omits any consideration of foreign trade. Later, the foreign trade aspects of the model, as shown in Diagram 2, will be discussed. The numbers in brackets in the following paragraphs refer to the stages in the sequence of calculations shown in the diagrams.

2. Closed economy

5.61. The starting point in Diagram 1 is the assumed level of gross domestic product based on the growth rate planned from the base year to the target year (1). This value and all the other demand and output levels will be expressed in constant base year prices. This total must be split into the four main components of demand - private consumption, public consumption and capital formation in both inventories and fixed capital (2). These are initial estimates, which will be subsequently amended in later calculations, and can be based on observations of past trends in the shares of the four components in the total. Better estimates may be available from a macro-economic model and in both cases it may be necessary to allow for any effect which broad socio-economic policy may have on aspects such as the balance between private and public consumption, or the balance between consumption and investment.

5.62. These four figures of demand by each sector must be disaggregated to give vectors of demand for each commodity group in the model (3). The details of private consumers' expenditure can be estimated by a model incorporating price and income elasticities and an initial assumption about the relative prices of commodities in the target year. The

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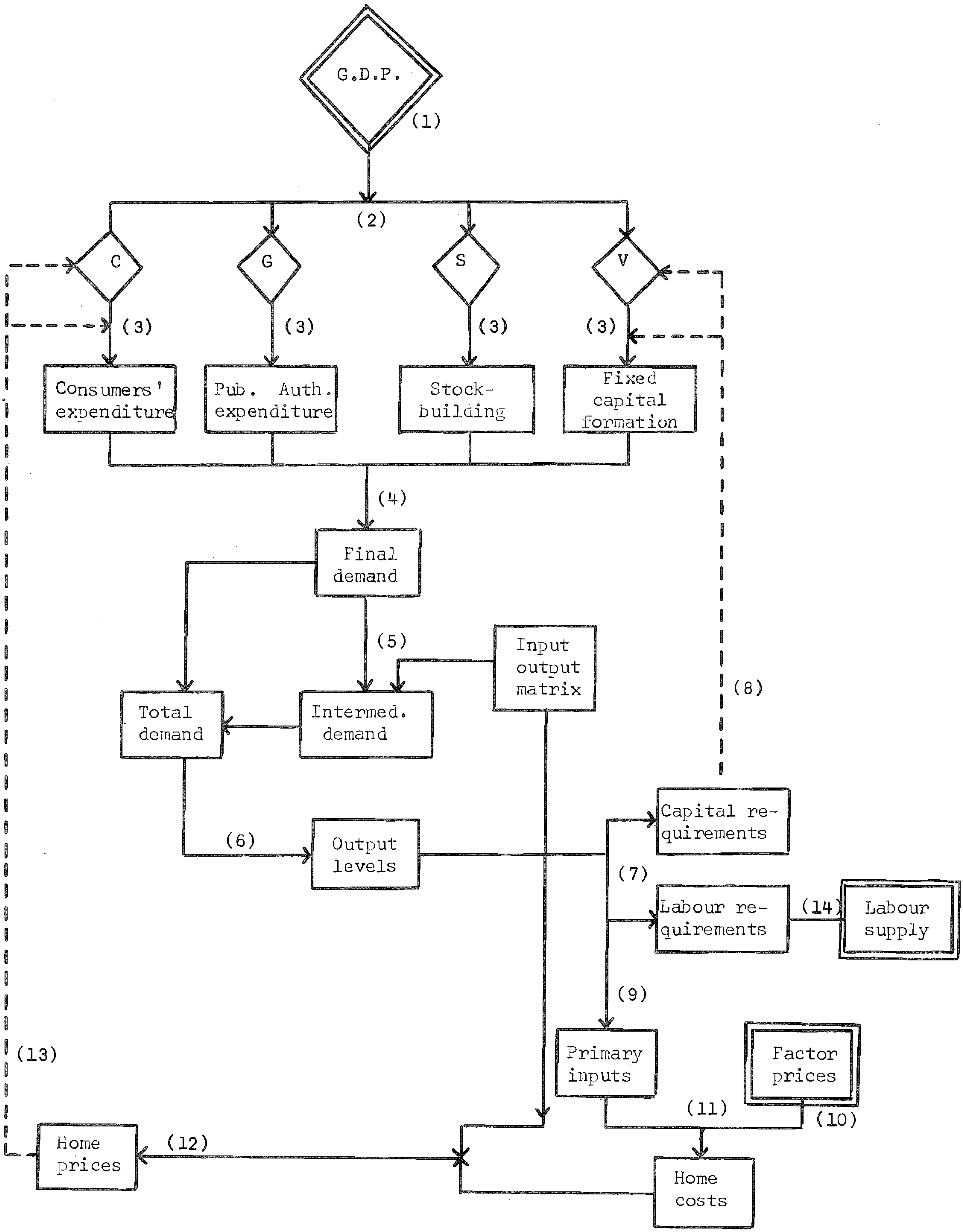
17/ See Department of Applied Economics, Cambridge University, Exploring 1972, Volume 9, A Programme for Growth, (Chapman Hall, 1970).

Diagram 5.1. Input-output model - closed economy



Note: See notes to Diagram 5.2.

classification of consumers' expenditure used in such a model may well have been the normal national accounting classification analysing the purpose, or object, of final consumption expenditure. The requirement in an input-output model is the same as in a full matrix presentation of the national accounts i.e. a commodity classification of expenditure. It will be necessary in this case to convert the classification from purposes to commodities (see S.N.A. paragraphs 2.33-2.36). The details of public consumption can be obtained from forecasts of the functional breakdown of expenditure made within government. An initial estimate of the commodity composition of investment demands is also required but this will be adjusted and improved during the calculations of output and capital requirements.

5.63. The initial estimates of the final demand for commodities are now completed (4) and can be combined with the input-output coefficient matrix, A, to yield estimates of the level of intermediate demand (5) and total required output levels (6) using the Leontief inverse, and equation (2.6).

5.64. When the output levels of industries are known the capital and labour requirements can be calculated (7). The former was discussed in the previous section where it was noted that it was possible to include a capital input-output matrix in the basic equation or alternatively to estimate investment demand using a production function approach separate from the main equation. Labour requirements can be estimated by using a production function approach or by a detailed study of labour productivity in each industry. The latter facilitates an analysis of labour requirements by type of skill but it is necessary to enforce consistency in the estimates of labour and capital requirements which would come automatically from a production function.

5.65. If investment demand is not being calculated using capital input-output matrices this is the stage when the initial estimates of investment demand will be revised in the light of the calculations of capital requirements based on output levels (8). The cycle of operations described so far will then be repeated to give a third estimate of investment demand and it will be necessary to repeat this cycle until a convergent solution is reached.

5.66. Finally, it is necessary to introduce prices into the model. All the previous calculations of demand and output have been expressed in constant base year prices in order to measure changes in the volume of output. It is also necessary to make forecasts about the levels of prices in the target year, in order to allow for changes in the relative prices of commodities and of factors. The calculations of capital and labour requirements will yield estimates of physical factor input coefficients in the target year (9). Estimates will also be made of the rates of payments per unit of factors (10) and these two estimates can be combined to give estimates of primary input coefficients (11). These estimates of home costs are used with the commodity input matrix, A, to calculate the prices of commodities using equation (5.2)(12).

5.67. These prices will be expressed as indices with the base year equal to unity. If it is felt to be difficult to estimate the likely absolute increase in factor prices (i.e. to predict the amount of inflation) this does not present a serious problem providing that an estimate can be made of any changes in relative factor prices. These estimated relative factor prices can be expressed as indices of the base year prices and used to estimate relative commodity prices in the target year. The absolute level of prices, although of some interest, is not vital to the working of a model such as this but changes in relative prices are important.

5.68. The final link of the diagram is to replace the initial estimates of relative commodity prices which were used to determine the details of consumers' expenditure by the calculated relative prices (13) and thus to adjust the initial estimates of final demand.

5.69. The final solution of this model will produce estimates of the following: the demand for commodities from each of the four sectors of final demand; intermediate demand for commodities; commodity output levels; labour and capital inputs; and the prices of commodities either relative or absolute levels.

5.70. It will finally be necessary to compare the estimated demand for labour with expected size and composition of working population (14). This may indicate that the chosen growth rate for the economy will require more labour than will be available or, alternatively, that it would result in an excessively high level of unemployment. If the labour requirements have been analysed in some detail the model may produce a set of requirements for labour skills which it is felt is so different from the existing pattern that adequate training and re-training could not be achieved in the span of years available. In these three cases it is then necessary to repeat the calculations with a different assumed growth rate until a satisfactory balance has been achieved between the supply and the demand for labour.

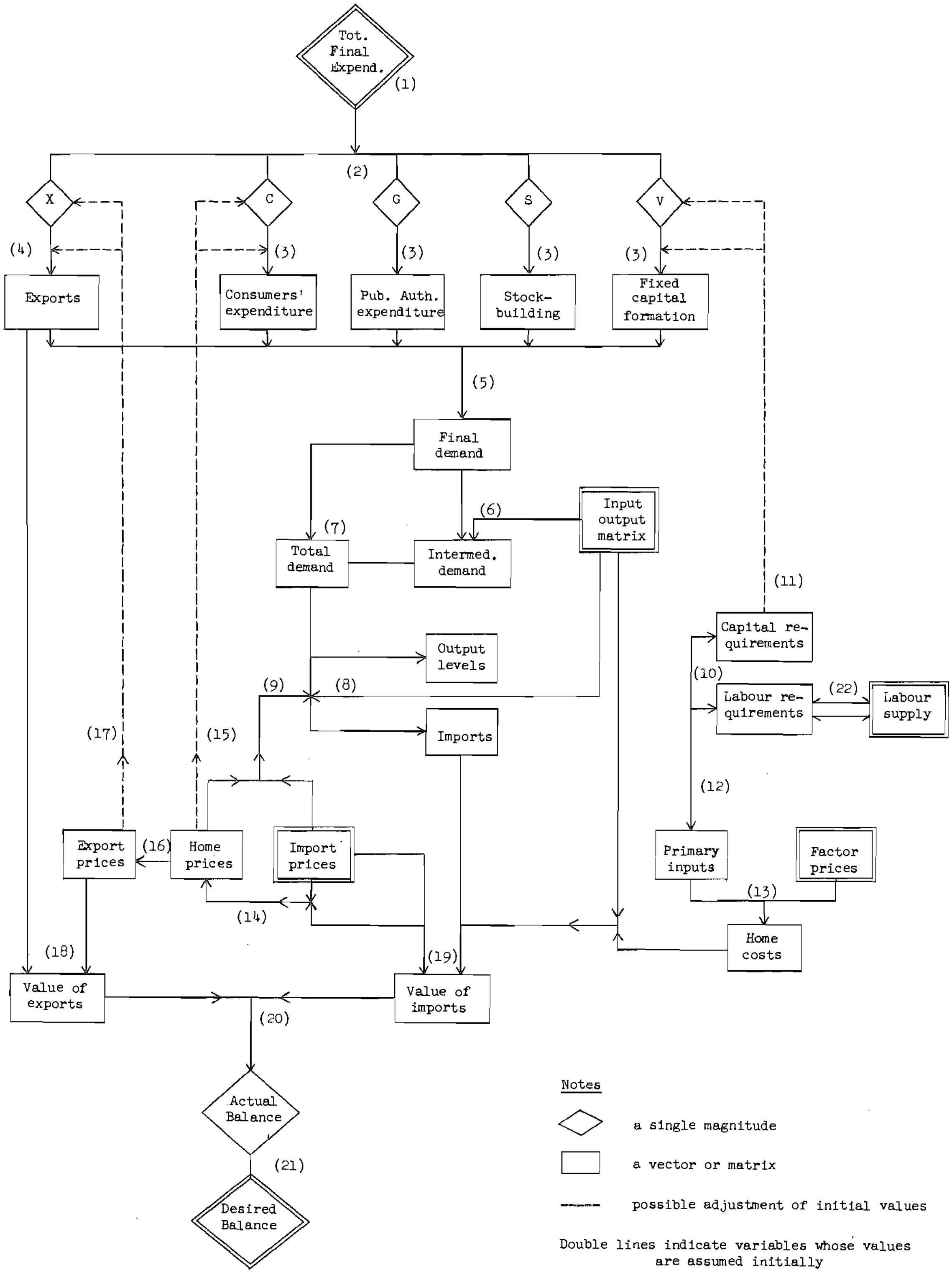
5.71. This solution has been described as a sequence of calculations, certain parts of which may be repeated a number of times as a convergent solution is reached. The alternative to this iterative solution would be the simultaneous estimation of all equations linking all the endogenous variables. This will be preferable providing the relations between the variable can be expressed in such a way as to make this possible. The choice between the two approaches, or a compromise, will largely depend on the exact nature of the relationship specified between some of the variables.

3. Economy with foreign trade

5.72. The illustrative model can now be extended to include foreign trade. There are various ways in which imports and exports can be incorporated into an input-output model but it is not intended to discuss these variations in the present volume.[18/] Diagram 5.2 serves to outline the relationships which must be incorporated when dealing with foreign trade although it does not specify the details of these relationships. Much of the sequence of calculations is the same as in the closed economy model in Diagram 5.1 and we shall mention only the additional items.

18/ For a good review of the various methods see Lecomber: "Input-Output and the Trading Economy" in Input-Output in the United Kingdom edited by W.F. Gossling, (Frank Cass, 1970).

Diagram 5.2. Input-output model with foreign trade



5.73. An initial estimate of total exports and their commodity composition must be made (4). This may be a simple extrapolation of past trends or it may be based on more sophisticated methods based on forecasts of the pattern of world trade and allowing for shifts in relative home and foreign price levels. In either case, the initial estimates are likely to be revised later.

5.74. Imports enter into the sequence of calculations at the stage when the total demand for commodities has been estimated (7). Methods of using input-output tables (8) to estimate the proportions in which these demands will be met from domestic production and imports were discussed in Section C of this chapter. It is often desirable to make the allocation of total supplies between domestic production and imports depend on the relative levels of home and foreign prices. The Cambridge U.K. model, for instance, includes a set of about 80 regression equations in which the level of imports of each of the commodities is dependent on the level of demand by final consumers and/or the main purchasing industries and the price level of imports relative to domestic prices. In many cases other explanatory variables peculiar to the commodity in question are also used.[19/] At this stage in the sequence of calculations these equations can be used to determine the levels of imports and domestic production of each of the commodities (9).

5.75. The calculation of factor requirements (10), the feedback to initial estimates of capital formation (11) and the determination of home costs (13) are the same here as in the closed economy model. When estimating the levels of home commodity prices it is necessary to allow for the fact that these should now depend on the prices of imported inputs in addition to primary inputs (14). This can be achieved by using the methods described in Section B of this chapter using equations (5.4) or (5.7).

5.76. The predicted levels of export prices will be usually very similar

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19/ See Department of Applied Economics, Cambridge University, The Determinants of Britain's Visible Imports, 1949-66, Vol. 10, A Programme of Growth, (Chapman and Hall, 1970).

to those of domestically produced goods sold at home (16). A possible reason for some difference would be if export prices had been observed in the past to move differently from domestic prices due either to a different product-mix or different competitive pressures. In some countries prices of exports have been found to rise less rapidly than those of domestically produced goods sold at home. When the levels of export prices have been predicted it may be necessary to revise the initial estimates of the commodity composition of total exports (17).

5.77. Finally, the open economy model has to deal with the balance of payments. The estimates of volume of imports and exports must be combined with the predicted price levels to give estimates of the value of trade in target year, rather than base year, prices (18, 19). All trade in marketed goods and services will be covered by these calculations but separate estimates will be required of a few other items in order to complete the current account transactions in the balance of payments . It will be necessary to obtain estimates of consumers' expenditure abroad, foreign consumers' expenditure within the economy, government receipts and expenditures and of income payments in both directions arising out of either employment or ownership of capital. When these are included an estimate will be available of the external balance on current account (20) which can then be compared with the desired balance (21). This will be arrived at after due consideration of the likely capital account transactions and level of reserves, both actual and desired.

5.78. This open economy model faces two constraints. Firstly, the supply of labour, as in the closed economy model (22), and, secondly, the balance of payments. If either of these constraints are not met it will be necessary to compute the model again assuming a different overall growth rate. Experience suggests that such a re-computation will be with a lower, rather than a higher, growth rate since in many cases one of the aims of such a model is to explore the feasibility of obtaining a higher growth rate than is currently prevailing in the economy.

4. Some results

5.79. The use of an input-output model for an open economy can be illustrated by looking briefly at some of the results of the Cambridge U.K. model referred to in the previous section. The base year of the projection was 1966 and the model was used to examine the implications for the U.K. economy of a certain growth rate between 1966 and 1972. The model devoted considerable attention to foreign trade and can thus be used to determine the balance of payments implications of various growth rates.

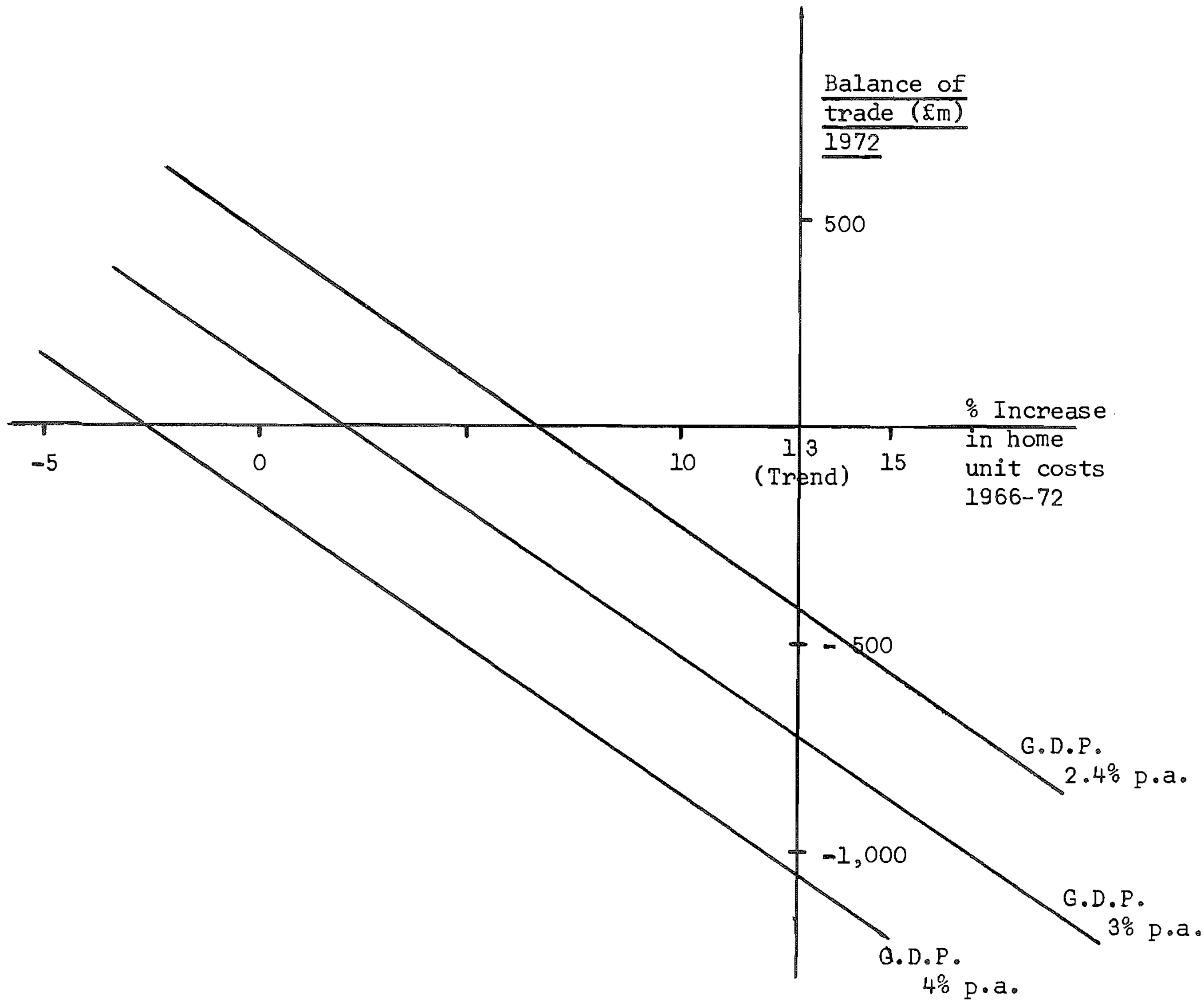
5.80. The full details of this projection of the model are to be found in the volume referred to in the previous section on page 123. Here reference will be made only to the balance of payments aspects.

5.81. The estimated relationship between the balance of payments and the level of home unit costs in the target year for three different growth rates over the projection period is shown in Diagram 5.3.

5.82. The initial assumption was made that the increase in home unit costs during the projection period would be in line with their past trend rate of increase. The past trend in the increase in foreign prices was also projected forward to 1972. The model showed that an annual growth rate in gross domestic product of 3% would result in a deficit of £715 million in the balance of trade in goods and services. It was then shown that equilibrium in the trade balance could be obtained if home unit costs in 1972 were only 2% above their 1966 level, whereas the projection of past trends suggested that the likely level in 1972 would be 13% higher than in 1966.

5.83. Given the prevailing rate of inflation in the U.K. and in other economies, such a small increase in costs was unlikely. The model was then estimated for a growth rate of G.D.P. of 2.4% - a rate significantly less than that prevailing over the previous decade. Even with this low growth rate the model suggested a deficit of £400 million with the trend value of home unit costs and that a deficit could only be avoided if the increase in home unit costs could be restricted to about 6%.

Diagram 5.3. Projected relationship between balance of payments and home unit costs, U.K. model



Source: Department of Applied Economics, Cambridge University, Exploring 1972 and unpublished results of calculations.

5.84. When an annual growth rate of 4% in G.D.P. i.e. significantly higher than the past trend was examined, the model showed that a deficit would only be avoided if home unit costs in 1972 could be at a level of 2% below the actual 1966 level.

5.85. The implications drawn from this study were that, since a sufficiently rigorous prices and incomes policy was unlikely to be instituted, serious consideration should be given to the devaluation of sterling. It seemed that only in this way could the level of home unit costs relative to foreign prices be brought sufficiently below the projected trend value and thus enable the U.K. economy to achieve even moderate economic growth and avoid a balance of trade deficit.

5.86. The actual outcome in the first five years of this six year projection period has shown that from 1966 to 1971 home unit costs relative to foreign prices have grown at the previous trend rate although both have risen faster over this period than their previous trend rate. From 1966 to 1971 the economy grew at 2.2% per annum which from Diagram 5.3 would suggest a deficit of about £300 million. The fact that a surplus of about £600 million was achieved in 1971 can be explained by the 14% devaluation of sterling in 1967 and by the fact that in 1971 the economy was operating below full capacity.

5. Projection of input-output coefficients

5.87. It will be useful to conclude this discussion of planning models by summarising the role which is played by input-output tables in such models. The primary function of the input-output table is to provide an estimate of the intermediate demand for commodities required to meet the postulated level of final demand. Secondly, the table will be used, to a greater or lesser extent, to make the division between imports and home-produced goods. The third role of the input-output table is in the determination of price levels. These applications of input-output have been treated separately in earlier sections of this chapter and need no further discussion.

5.88. It is appropriate, however, to consider the question of obtaining an estimate of the input-output coefficients for the target year. It was shown in Chapter III, Section M that input coefficients are not stable over time and it is obvious that base year coefficients will yield inaccurate results if applied to the target year.

5.89. The problem of projecting input coefficients is increased since many countries have only a small number of tables relating to previous years and these may not be strictly comparable one with another due to changes in classification or having been compiled by different organisations. If a sufficient number of past observations were available, it would be possible to measure the time trend in each of the coefficients and thus to project the coefficients for the required number of years into the future. Typically, however, a sufficient number of past tables are not available and this approach cannot be applied.

5.90. The extreme situation arises when there is only one observed table for a previous year based on full information and one (or more) estimated using the R.A.S. methods described in Chapter III, Section L. There it was shown that the input coefficient table for the current year, A_t, could be estimated from an earlier table say, A_{t-5}, using row and column multipliers, r and s, expressing, respectively, substitution and fabrication effects. Thus:

$$A_t = \hat{r} A_{t-5} \hat{s} \tag{5.16}$$

5.91. If an estimate of a future table, say, A_{t+5}, is required, this could be obtained by a simple projection of the row and column multipliers based on the current matrix:

$$A_{t+5} = \hat{r} A_t \hat{s} \tag{5.17}$$

If the projection period is different from the time period between the base and current years, the r and s multipliers must be changed. Thus if a projection for ten years ahead was required we would have:

$$A_{t+10} = \hat{r}^2 A_t \hat{s}^2 \tag{5.18}$$

5.92. The limitations of this approach will be very apparent in view of the earlier discussion of the simple R.A.S. method. No exogenous information is included here about either the past movements in coefficients or possible future changes, and errors in the multipliers are projected into the future.

5.93. A more typical situation is where it is necessary to make estimates of future coefficients on the basis of perhaps only three past observations of the coefficients. Whilst it is obviously possible to fit a time trend to only three observations, the margin of error is likely to be so great that the results must be interpreted and used with very great care. This is particularly the case with any coefficients which exhibit a cyclical movement since it will be very difficult to distinguish the trend from the cycle. Furthermore, the exercise will be hindered by possible, unknown, errors of measurement in the basic data.

5.94. It is desirable wherever possible to incorporate exogenous information on particular coefficients. As was suggested earlier (paragraphs 3.132-3.133) there will be a number of coefficients for which data is available annually. A careful study of past movements in such coefficients must be carried out and a future projection made.

5.95. Tentative projections can be made of the other cells based on the few observations which are available. These projections should be checked with industrial experts wherever possible and, if necessary, amended. Such liaison between those centrally involved in the projection exercise and people directly involved in the industries under consideration should considerably improve the accuracy of the projections.

5.96. It was noted in paragraph 3.134 that it is useful to know which are the cells in the input-output matrix which have the greatest effect on the accuracy of the results. This is as relevant to a projection exercise as it is to an up-dating exercise. Estimates of intermediate demand will be more sensitive to changes in some cells than to changes in other cells. If the more important cells are identified it will be useful to devote resources to a study of their movement over time and thus to improve the quality of the projection.

5.97. With the passage of time the number of observations of input coefficients is increasing in all countries, the accuracy of the data will be improving and the range of industrial statistics will be extending and it should, therefore, be possible gradually to improve the degree of accuracy with which one can estimate future coefficients. Less reliance will need to be placed on crude R.A.S. extrapolations and on fewer occasions will predictions be made based on a very limited number of past observations. In more cases it will be possible to study a time series of coefficients, to observe how this changes and to explore some of the reasons for the observed changes. In all cases, even with maximum data, consultation with industrial experts should considerably improve forecasts of coefficients.

F. Dynamic input-output

5.98. In recent years increasing attention has been devoted to dynamic input-output models and the distinction between static and dynamic systems must be made clear. We began in earlier sections of this chapter with a simple input-output model in which the demand for capital goods was treated as exogenous. In Section D we introduced capital matrices and thus allowed the level of investment demand to be determined within the model. This acknowledges the link between one year and another but does not make the model a dynamic one. Such a model remains a static model since it concentrates on an examination of the economy in the terminal year of the target period. It may be noted from such a model that the demand for capital goods will generate a demand for output in the year or, if the time lags are long enough, years before the target year. Apart from this the intervening period will be ignored since the model is set up for optimisation at a single point in time.

5.99. The essential distinction of a dynamic model is that it traces the path of the economy from the base year to the target year in such a way that it becomes possible to optimise the pattern of future growth given some objective function, rather than focussing attention solely on the final year of the target period. In a static model no attention is paid to the actual path of the main variables in the economy which are simply assumed to adjust to the required levels.

5.100. A dynamic model in its purest form can be written:

$$\dot{q}_t = A\dot{q}_t + K\dot{q}_t + \dot{c}_t \tag{5.19}$$

where $\dot{q}_t$ is the rate of change of output with respect to time (dq/dt). Such models are theoretically appealing but computational and estimation problems confine the practical applications to models of a simpler type looking at a series of discrete but related time periods.

5.101. In this case we shall have a series of equations in the form of equation (5.10) relating to each of the years in the planning period of t years:

$$\begin{aligned} q_1 &= Aq_1 + K\Delta q_1 + c_1 \\ q_2 &= Aq_2 + K\Delta q_2 + c_2 \\ &\vdots \\ q_t &= Aq_t + K\Delta q_t + c_t \end{aligned} \tag{5.20}$$

5.102. These equations express the demand for capital goods in a very simple manner by relating it to the future growth in output using the capital input matrix, K. In this way the link between successive time periods is established and it becomes possible to examine the effect which a particular level of demand in a certain year has on the output levels of other industries in preceding years as well as in the current year.

5.103. Since $\Delta q_t = q_{t+1} - q_t$, it is possible to re-write equation (5.20) in its general form as:

$$q_t = Aq_t - Kq_t + Kq_{t+1} + c_t \tag{5.21}$$

5.104. If we re-arrange the terms in (5.21) and write $H = (I - A + K)$ we have:

$$
\begin{aligned}
H_1q_1 - K_1q_2 &= c_1 \\
H_2q_2 - K_2q_3 &= c_2 \\
&\vdots \\
H_tq_t - K_tq_{t+1} &= c_t
\end{aligned}
\qquad (5.22)
$$

5.105. If we are considering a planning period of t years it is convenient for discussion to exclude the effect on these years of output levels in subsequent years; we can then omit the term in q_{t+1} from the last line of equation (5.22). For simplification also, we shall assume that there is no technical change in the economy i.e. that the coefficients in A and K are constant. This enables us to omit the time subscripts from H and K. We thus obtain the following solution for q_t:

$$q_t = H^{-1} c_t \qquad (5.23)$$

5.106. From the next to last line of equation (5.22) we obtain:

$$q_{t-1} = H^{-1} c_{t-1} + H^{-1} Kq_t \qquad (5.24)$$

If we substitute the value of q_t from (5.23) into (5.24) we obtain a solution for q_{t-1}:

$$q_{t-1} = H^{-1} c_{t-1} + H^{-1} KH^{-1} q_t \qquad (5.25)$$

5.107. Similarly, from the previous line of (5.22) we obtain:

$$q_{t-2} = H^{-1} c_{t-2} + H^{-1} Kq_{t-1} \qquad (5.26)$$

and substituting (5.25) into (5.26) we have:

$$q_{t-2} = H^{-1} c_{t-2} + H^{-1} KH^{-1} c_{t-1} + H^{-1} KH^{-1} KH^{-1} c_t \quad (5.27)$$

5.108. This stepwise solution can be continued back through equation (5.22) until a solution for the first year, and thus all years, of the planning period has been obtained. Each of these solutions will be similar to equation (5.27) but will contain increasingly more terms.

5.109. It must be remembered that each of the terms q and c in the above equations are vectors and not single numbers; they represent respectively the output levels of a number of commodities and the final consumer demand for those commodities. In this way it is possible to determine the sequence of annual output levels in each industry that would enable the economy to yield the sequence of final demands given on the right hand side of equation (5.22).

5.110. The dynamic input-output system was developed by Leontief[20] who has given a more formal exposition using a full matrix presentation, in which he illustrates the application of the dynamic inverse to the U.S. economy.

5.111. In a static input-output system it is possible to calculate the direct and indirect output requirements of each commodity of one unit of final output of a given commodity. In a similar way the dynamic inverse can be applied to calculate the requirements of a given amount of final output not only in the current year but also, through direct and indirect capital requirements, in all preceding years.

5.112. It is useful to quote a specific example of Leontief's calculations using the dynamic inverse, in which he shows the output levels in other industries in preceding years required to yield \$1,000 output of machinery in the final year. The output of metals required in the final year is \$120, the amount which one would calculate using the static inverse matrix, $(I - A)^{-1}$. In the previous year \$55 output of metals will be required and in preceding years the requirements decline quite rapidly

[20] Leontief, W.W., "The Dynamic Inverse" - in Contributions to Input-Output Analysis, edited by A.P. Carter and A. Brody (North Holland, 1970).

but amount to \$5-\$10 four years before the current year and are still significant seven years earlier. This long period over which output is required reflects to a large extent the high metals content of capital goods; the demand for some other products falls to zero much earlier when requirements are traced back through past years.

5.113. This description of a dynamic input-output system is based on a very simple assumption about the demand for capital goods i.e. that this is determined by the growth in output in the year ahead and that capital goods produced this year will be in full production next year. Various ways in which this simple assumption can be qualified were discussed in Section D of this chapter (paragraphs 5.49-5.53). Introduction of such qualifications make the system more realistic but add considerably to the computational requirements of the dynamic inverse.

5.114. The dynamic system, such as that described above in equation (5.22), represents a very simple view of an economy. In order to go from this to a full dynamic model of an economy requires the introduction of separate vectors of final consumer demand, the relationship of imports and exports to the domestic economy, price equations etc. In other words what is involved is a model similar to that described in Diagram 5.2 which must be set up in such a way that each of the years in the planning period are linked together with the dynamic inverse. The model can then be estimated and balanced for each of the years and the time-path of the economy can be examined.

5.115. This is obviously a formidable task but a few such models have been set up in recent years. Among these are a model of the Israeli economy estimated by Bruno, Dougherty and Fraenkel[21/] which covers a planning period of 15 years divided into six sub-periods of $2\frac{1}{2}$ years. This model maximises an objective function of the discounted flow of private consumption over the planning period and the discounted value of the capital stock at the end of the planning period, the latter being included to take into account the contribution of the present generation to the

21/ Bruno, M., Dougherty, C., and Fraenkel, M.,"Dynamic Input-Output, Trade and Development" in Applications of Input-Output Analysis, edited by A.P. Carter and A. Brody (North-Holland, 1970).

welfare of future generations. Dynamic models incorporating a linear programming solution have been developed in Japan[22/] and Hungary[23/] in order to examine the optimal growth path of the economy.

5.116. Most of the attempts at developing dynamic models run into data and computational limitations which are often overcome by the use of quite highly aggregated sectors. Although many problems exist at present, dynamic models will generally be superior to static models which concentrate on a single target year and thus ignore any implications for the economy in intervening years of the chosen growth rate. Dynamic models overcome this by looking at the future growth path of the economy year by year and the data and computational limitations which at present hinder their practical application will doubtless become somewhat less of a problem in the future.

G. Index numbers

1. General

5.117. Index numbers of the quantity and price of the production, import and use of goods and services are required in dealing with many aspects of the management of a modern economy and such indices are, therefore, commonly compiled as independent series as well as forming an integrated part of the national accounts. Current attempts to establish an integrated set of price and quantity index numbers will fit both the series of national accounting in constant prices and the traditional indices of price and quantity into a single, coherent, articulated framework which will enhance the analytical usefulness and reliability of these series of data (E/CN.3/427 and E/CN.3/428).[24/]

5.118. The framework of national accounts at current and constant prices

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22/ Murakami, Y., Tomoyama, K., and Tsukui, J., "Efficient Paths of Accumulation and the Turnpike of the Japanese Economy", op.cit.

23/ Augustinovics, M., "A Twin Pair of Models for Long-Term Planning", paper presented to the Fifth International Conference on Input-Output Techniques, Geneva, 1971.

24/ _Official Records of the Economic and Social Council, Fifty-fourth Session, Supplement No. 2_, chap. V.

which is described in detail in the S.N.A. (see particularly Chapter 8), provides a basis for the decomposition of flows of goods and services into price and quantity components. Such flows relate to gross output, intermediate and final consumption, gross fixed capital formation, increase in stocks and imports and exports. It is desirable to have an index number system which provides price and quantity indices of these flows, in many cases distinguishing, in addition, the commodity composition of these flows.

5.119. It is, therefore, important that the national accounting framework should include an input-output table at constant prices among the flows which are to be decomposed in price and quantity elements. As was emphasised in Chapter I and Chapter III, Section N, the inclusion of the input-output table and its conversion to constant prices at the same time as other flows are being revalued will make for consistency between the series in various parts of the accounts, thus increasing the accuracy of the work.

5.120. The central role of the input-output table in the production account in the national accounting framework makes it an invaluable frame for purposes of determining the weights, base-year and current-year, of the system of index numbers, selecting interrelated samples for collecting integrated series of quantity, price and value indicators, and compiling linked and consistent series of index numbers of price, quantity and value. Though adequate basic data may not be available to compile the suggested input-output tables of index number series completely each year, it would be desirable to carry this work forward as far as is possible. In any event, it will be advantageous to use the input-output tables as the frame for purposes of compiling annual, and even quarterly, series of index numbers and for purposes of selecting samples and weighting in the case of all types of inquiries. The compilation of annual series of index numbers within the framework of input-output tables will, along with other things, furnish relatively current weights for price series and the basis for compiling monthly series of index numbers on the disposition of goods and services to domestic sources.

5.121. The use of input-output tables as a frame for compiling the system of index numbers should also be of assistance in filling in gaps in the available series of indicators.

2. Value added

5.122. One of the most important uses of an input-output table in the construction of index numbers is its role in the estimation of value-added at constant prices by the double-deflation method. The value-added in each industry can be obtained by subtracting the value of commodity inputs from the value of gross output and the sum of the value-added in each industry provides an estimate of gross domestic product from the output side. The estimation of value-added at constant prices thus involves revaluing the input-output table at constant prices and subtracting the sum of commodity inputs into an industry at constant prices from the value of its gross output also revalued at constant prices.

5.123. In terms of the presentation in the S.N.A. (Chapter 4, Annex), and assuming no difference between industries and commodities, the vector of value-added in each industry in year 0, y_o, can be written as:

$$y_o' = p_o' (I - A_o)\hat{q}_o \qquad (5.28)$$

The first term here, $p_o'\hat{q}_o$, measures the value of gross output of each industry and the second term, $p_o'A_o\hat{q}_o$, is the value at year 0 prices of the column totals of the intermediate inputs of the input-output table ($X = A\hat{q}$).

5.124. It is convenient to re-arrange the terms in equation (5.28) and obtain:

$$y_o = \hat{q}_o (I - A_o')p_o \qquad (5.29)$$

The vector of industry value-added in year 1 valued at year 0 prices can be written as:

$$y_{o.1} = \hat{q}_1 (I - A_1')p_o \qquad (5.30)$$

Here both gross outputs and inputs have been revalued at year 0 prices and subtracted to give value-added at constant prices.

5.125. The Laspeyres base-weighted index of value added can then be obtained by dividing (5.30) by (5.29):

$$\Lambda^* = \frac{\hat{q}_1 \, (I - A_1')p_o}{\hat{q}_o \, (I - A_o')p_o} \tag{5.31}$$

5.126. The method of measuring quantity changes in value-added by industry which is commonly used if an input-output table for the current year, 1, is not available, is the single-deflation method which consists of applying value-added weights of the base-year to single indicators of the different outputs. This form of index can be written as:

$$\bar{\Lambda}^* = \frac{\hat{q}_1 \, (I - A_o')p_o}{\hat{q}_o \, (I - A_o')p_o} \tag{5.32}$$

5.127. In general the assumption that the coefficient matrix has not changed will not be justified and this index is likely to produce an inaccurate result. This can be seen more clearly if equation (5.31) is written in expanded form:

$$\Lambda^* = \frac{\hat{q}_1 \, (I - A_o')p_o}{\hat{q}_o \, (I - A_o')p_o} + \frac{\hat{q}_1 \, (A_o' - A_1')p_o}{\hat{q}_o \, (I - A_o')p_o} \tag{5.33}$$

The first term in equation (5.33) is the single-deflation index given in (5.32), and the second term, therefore, measures the error in the single-deflation method.

5.128. It is thus seen that the single-deflation method will be inaccurate if the input-output coefficients have changed during the period in question. It is not suggested, however, that double-deflation methods are necessary for the compilation of the more frequent quantity indices since coefficients will not change significantly during the course of a year. For such indices single-deflation methods will be adequate.

5.129. When considering annual indices, even if $A_o \neq A_1$ it is possible that the error in the single-deflation method may not be large because positive and negative components may to some extent offset one another. Such compensating errors cannot be relied upon and whenever possible input-output tables at constant prices should be used to make estimates by double-deflation, although this method can, in some circumstances, produce anomalous results.

5.130. Paradoxical results may arise if some important input is used in much larger quantities than in the base year because of a considerable fall in its relative price; in this case the quantity index of value added may show a decline, even if both value added at current prices and the quantity index of gross output increase substantially. If a considerable decrease in the quantity of gross output takes place at the same time, value added at constant prices could even become negative. Since such unrealistic results are likely to be caused by one, or a few, major factors, it should,however, be possible to identify the reasons for them. A change of base year may be necessary if a considerable change in the price structure for either output or input is found to be the major reason.

5.131. If the indicators utilised in double-deflation are not sufficiently accurate, unrealistic quantity series of value added may also be obtained when input-output coefficients are high. For instance, if the quantity indicators utilised in respect of intermediate input and output are not representative, an increase in gross output which is actually due to a proportionate increase in intermediate input, may be reflected differently in the two indicators and may result in spurious changes in the quantity index of value added. Similar spurious changes in the price of quantity measures may, of course, result if unrepresentative price indices are used in respect of, or to deflate, gross output and intermediate input.

3. Productivity

5.132. An interesting and important use of input-output based index numbers is the study of changes in productivity. This can be achieved by comparing measures of value-added at constant prices with measures of primary input of factors at constant prices.

5.133. The decomposition of the value of labour and capital inputs into quantities and prices is a subject on which more work is needed before a set of index numbers for these factors could be included in any overall system of index numbers. Considerable experience has been accumulated in evaluating the value of wages and salaries in terms of employment and wage and salary rates but not in expressing the share of capital in value added in terms of the quantity of capital employed and a rate of return to capital.

5.134. It is reasonable to assume that such problems will be overcome and thus to examine the system of productivity indices described in the S.N.A. (paragraphs 4.113 through 4.129).

5.135. In Section B of this chapter we wrote the basic price equation for an input-output system, equation (5.1),as:

$$p = (I - A')^{-1} \bar{y} \qquad (5.34)$$

and we distinguished between the input coefficient of the value of labour and of capital by writing $\bar{y} = \bar{w} + \bar{r}$ in equation (5.2). In order to estimate a quantity series for factor inputs we must also distinguish between the physical input and the rate of return to each factor.

5.136. In order to do this we define a coefficient matrix, F, an element of which measures the quantity of one of the primary inputs required per unit of output of one of the commodities. This matrix relates primary inputs to commodity outputs in just the same way that the matrix A relates intermediate (commodity) inputs to commodity outputs. Further, let s denote a vector of the return per unit for the use of each primary input. Then we can write the basic factor cost equation (5.34) in the form

$$p = (I - A')^{-1} F's \qquad (5.35)$$

5.137. We can substitute the expression for p in equation (5.35) in the quantity index-number of value-added in equation (5.31) and obtain

$$\Lambda^* = \frac{\hat{q}_1 (I - A_1') (I - A_o')^{-1} F_o' s_o}{\hat{q}_o F_o' s_o} \qquad (5.36)$$

5.138. The required quantity index for primary inputs, Λ^{**}, will take the form

$$\Lambda^{**} = \frac{\hat{q}_1 F_1' s_o}{\hat{q}_o F_o' s_o} \qquad (5.37)$$

This index is obtained by valuing the physical factor inputs in both years ($\hat{q}$ F') at the rates of return of the base year.

5.139. We are now in a position to derive the indices of productivity, Λ^{***}, by dividing (5.36) by (5.37)

$$\Lambda^{***} = \frac{\hat{q}_1 (I - A_1') (I - A_o')^{-1} F_o' s_o}{\hat{q}_1 F_1' s_o} \qquad (5.38)$$

From this we can see that a change in productivity is composed of changes in intermediate technology, A, and changes in factor technology, F, both of which contribute towards the overall change.

5.140. Finally, it is possible to construct indices to measure separately these two components of productivity change. Firstly, if we assume there has been no change in intermediate technology, i.e. that $A_o = A_1$, we can measure that part of the change in productivity which is due to a change in factor technology and, similarly, by assuming that $F_o = F_1$ we can measure that part of the change of productivity which is due to a change in intermediate technology.

5.141. These two indices of partial productivity can thus be derived from equation (5.38) to give:-

$$\Lambda^{***}(A_o = A_1) = \frac{\hat{q}_1 F_o' s_o}{\hat{q}_1 F_1' s_o} \quad (5.39)$$

$$\Lambda^{***}(F_o = F_1) = \frac{\hat{q}_1 (I - A_1')(I - A_o')^{-1} F_o' s_o}{\hat{q}_1 F_o' s_o} \quad (5.40)$$

5.142. An examination of equations (5.38), (5.39) and (5.40) shows the index of total productivity is the product of the two partial productivity indices i.e.

$$\Lambda^{***} = \Lambda^{***}(A_o = A_1) \times \Lambda^{***}(F_o = F_1) \quad (5.41)$$

5.143. It will be noted that all the above indices, both of value added and of productivity, are Laspeyres base-weighted indices. It is, of course, equally possible to construct Paasche current-weighted indices and the corresponding equations are to be found in the S.N.A. (Chapter 4, Annex).

5.144. It will be useful to conclude this section by looking at a set of productivity indices calculated for the U.K. using the above methods;[25] these results are shown in Table 5.13.

5.145. The output indices measuring value added at constant prices were calculated by the double-deflation method in equation (5.31). These were obtained using 35 sector input-output tables for 1954 and 1963, the latter having been revalued to 1954 prices. As was noted above certain conceptual problems are involved in obtaining quantity and price indices for factor inputs i.e. the elements of matrix F and vector s. In the U.K. calculations simple measures were taken of physical factor input. These

25/ Armstrong, A.G., Volume 12 in A Programme for Growth by the Department of Applied Economics, Cambridge University, (Chapman and Hall, forthcoming).

Table 5.13. Output and productivity indices, U.K. 1963 (1954 = 100)

Industry	Value added at constant prices	Factor input	Productivity		
			Total	Partial	
				$A_o=A_1$	$F_o=F_1$
	Eq. 5.31 (1)	Eq.5.37 (2)	Eq.5.38 (3)	Eq.5.39 (4)	Eq.5.40 (5)
Agriculture	123	109	113	115	98
Mining	83	79	105	112	94
Food, drink, tobacco	121	106	114	120	95
Chemicals	200	123	162	126	128
Metal manufacture	120	113	106	102	104
Shipbuilding, engineering and electrical goods	135	115	117	115	102
Vehicles	173	113	152	127	120
Textiles, clothing and leather	115	85	135	114	118
Other manufactures	124	114	108	114	95
Construction	128	126	102	100	102
Gas, electricity, water	157	129	122	119	102
Transport, distribution, other services	126	122	104	105	99
All industries	128	115	111	109	102

were man-hours of employment and gross fixed capital stock at constant replacement cost. The rate of return per unit of factor input was obtained by dividing income from employment and profits, respectively, by the total physical input of labour and capital.

5.146. The results in Table 5.13 show that an output increase of 28% was achieved with an increase in primary inputs of 15% thus giving an increase in productivity of 11%. The productivity indices for individual industries range from 102 for Construction to as high as 162 (i.e. over

6% per year) for chemicals. The last two columns of Table 5.13 show the results of calculating the two partial productivity indices (5.39) and (5.40).

5.147. The value of the indices in column 4 show the part of the productivity increase in each industry which is due to changes in factor technology i.e. intermediate technology is assumed to be unchanged. It is noticeable that for the economy as a whole, as well as for most of the individual industries, changes in factor technology are more important than changes in intermediate technology as a source of productivity change.

5.148. These results, which are based on equations (5.38), (5.39) and (5.40), are interesting and demonstrate the central role of the input-output table as a basis for the study of technical change. However, as noted above, more work is needed in this field before quantity and price indices of primary inputs can be incorporated into an integrated overall system of index numbers.

ANNEXES

Annex I

NOTATION

X' - is the transpose of X; i.e. with rows and columns interchanged.

$\hat{x}$ - is a matrix formed with the elements of the vector x as its leading diagonal.

$\bar{x}$ - is the vector x expressed in coefficient form.

Matrices

A - commodity x commodity coefficient matrix.

B - commodity x industry coefficient matrix; the values in the absorption matrix expressed as coefficients.

C - product-mix matrix, the columns of which show the proportions in which a particular industry produces various commodities.

D - market-share matrix, the columns of which show the proportions in which various industries produce the total output of a particular commodity.

E - industry x industry coefficient matrix.

F - primary input coefficient matrix, measuring the quantity of each primary input used per unit of output.

H - $= I - A + K$.

I - the unit matrix, with 1's in the leading diagonal and 0's in all other cells.

K - capital input coefficient matrix, measuring the stock of each commodity held per unit of output in each industry

M - make-matrix, recording the values of commodities produced by industries.

S - capital stock matrix measuring the value of the stock of each commodity held in each industry.

X - absorption matrix recording the value of the purchases of commodities by industries.

W - commodity x commodity flow matrix recording the value of the purchases of commodities by commodities

Z - industry x industry flow matrix recording the value of the purchases of industry outputs by industries

Vectors

b - the proportion of total home (intermediate + final) demand supplied by imports.

c - consumers' demand for commodities.

d - the proportion of total home (intermediate + final) demand supplied by domestic production.

e - final demand for the outputs of industries.

f - final demand for commodities (= c + v + x).

g - outputs of industries.

h - home final demand for commodities (= c + v = f - x).

i - the unit vector.

m - imports of commodities.

p - prices of commodities.

p_h - prices of domestically-produced commodities.

p_m - prices of imported commodities.

p_n - prices of input commodities.

q - outputs of commodities.

r - profit incomes paid by industries (also, row multipliers in R.A.S. method).

s - return per unit of primary input (also, column multipliers in R.A.S. method).

u - row totals of input-output matrix (in R.A.S. method).

v - demand for commodities for capital formation (also, column totals of input-output matrix in R.A.S. method).

w - wage incomes paid by industries.

x - exports of commodities.

y - value of primary inputs into industries (i.e. factor incomes paid by industries).

z - intermediate demand for commodities.

Annex II

NATIONAL PRACTICES IN COMPILING INPUT-OUTPUT TABLES

1. The tables set out in this annex furnish a comparison of national practices in compiling and issuing input-output data. They are based on publications, mimeographed documents and information from national statistical authorities that were available to the Statistical Office of the United Nations.

A. Table 1

2. Table 1 shows the countries who have compiled input-output data and the years for which they have done this. It also indicates which of the input-output tables of each country are analysed in this annex. In general, the latest, most complete input-output data was selected for this purpose.

3. A number of countries - somewhat over 20 developed market economies, about the same number of developing market economies and practically all centrally planned economies - compile input-output data periodically. It should be emphasized that the input-output tables of the centrally planned economies concern the supply and use of goods and material services only whereas those of the market economies cover all goods and services. The intervals of time between their input-output tables range from about ten years to five years to one year. Most countries prepare the tables for years for which comprehensive, detailed statistics are available. Such data are available at about ten- or five-year intervals of time only in the case of a number of the countries and annually in the case of some of them. A number of the countries compiling annual input-output data do this by using limited annual statistics to extrapolate their comprehensive base-year data.

B. Table 2

4. Table 2 furnishes information on the type of basic input-output tables countries compile and the classifications of producers and commodities, of final dispositions of commodities and of primary inputs that they use.

1. Scope and types of input-output tables

5. The table indicates that the comprehensive input-output matrices issued by countries consists of wings on the final uses of commodities and on the primary inputs into production as well as an inner sub-matrix on the intermediate inputs into production.

6. The points of departure for preparing these series of data usually are figures on the cost-structure of establishments, classified according to their kind of industrial activity, on the kinds of commodities which they produce domestically and are imported, and on the domestic disposition and export of these commodities. As establishments may produce secondary products, the original basic input-output data are, in the case of many of the countries with market economies, classified in the following manner: primary inputs according to kind of industry; gross output and intermediate inputs according to kind of commodity and kind of industry; and final dispositions according to kind of commodity. This amounts to an input-output table with kinds of commodities in the rows and kinds of industries in the columns, that is a commodity x industry tabulation. Whether the comprehensive input-output matrices that are tabulated in column 1 are, in fact, commodity x industry, industry x industry, commodity x commodity or a mixture of the foregoing depends on the extent to which and the manner in which secondary products and the associated intermediate and primary inputs have been transferred to the activity in which they are characteristically produced. These questions are dealt with in table 3 of this annex.

7. The comprehensive input-output tables of the centrally planned economies are industry x industry tables that are often directly compiled from special data gathered from enterprises.

8. As column 2 of table 2 indicates, a few countries supplement their comprehensive input-output matrix by issuing a table in which the gross output of establishments is classified according to kind of commodity and the kind of industrial activity of the establishment. This table furnishes valuable information on the kind and amount of secondary, as well as characteristic, production of the establishments classified in each kind of industrial activity for purposes of describing the structure of production and for purposes of the algebraic transfer of the secondary products and the associated cost-structure to the industries where they are characteristically produced.

2. Classifications of commodities and industries

9. Columns 3 and 4 of table 2 describe the size of the comprehensive input-output tables issued by countries. It may be noted that except for four tables - those of Belgium, Canada, Fiji and Togo - the inner (absorption) sub-matrix of these tables

consists of the same number of rows and columns. While it is feasible and useful for certain purposes to classify data on commodities in greater detail than data on industries, this interferes with the inversion of an input-output table.

10. The number of rows and columns of the inner sub-matrix of the comprehensive tables analysed in table 2 range from lows of 10 or less to highs of 100 or more. There are somewhat over 20 small (under 30 rows and columns) tables, about 25 moderate (30 through 79 rows and columns) tables and 12 large (80 and over rows and columns) tables. All of the small tables have been compiled by developing countries, primarily because of the inadequacy of the available data for compiling larger tables.

3. Classifications of final use

11. Columns 7 through 15 of table 2 indicate that the categories of the final disposition of commodities that market-economy countries use in their input-output tables correspond, in general, to those of the United Nations System of National Accounts (SNA). Because of inadequacies in the available basic data, a number of developing countries have found it necessary to combine private and public (government) final consumption, or even all categories of final domestic demand. While the definition of the classes of final demand that centrally planned economies use in their input-output tables differ from those of the SNA - for example they relate to goods and material services only and the outlays on these items by private units of the non-material sphere are included in private consumption - the distinctions made between these classes are similar to those drawn in the SNA.

12. As may be noted from the footnotes to table 2, the input-output tables of some countries include classifications of certain of the categories of final demand - for example private consumption according to type of household, public consumption according to level of government or according to purpose, gross fixed capital formation according to type of fixed assets or according to kind of owner or user. The classification in the USSR input-output table of gross fixed capital formation into that for expansion, that for replacement of wornout fixed assets and that for replacement of losses in fixed assets, is particularly noteworthy.

4. Classifications of primary inputs

13. Columns 16 through 25 of table 2 show that the categories of primary inputs recommended in the SNA - compensation of employees, depreciation, net indirect taxes and operating surplus - are followed in the input-output tables of a number of

market economy countries. Some of these countries find it desirable and feasible to use a more detailed classification of primary inputs - for example sub-division of compensation of employees into wages and salaries and employers' social security contributions or classification of net indirect taxes into subsidies and indirect taxes. Other of the countries do not have sufficient basic data in order to make any sub-division of value added.

14. The common classification of primary inputs in the input-output tables of the centrally planned economies is wages and salaries, turnover taxes and operating surplus of the enterprises of the material sphere of the economy. Their depreciation is included in intermediate consumption and their social security contributions are included in operating surplus. The operating surplus of the enterprises also includes the value of the non-material services that they use.

C. Table 3

15. Table 3 concerns the practices followed in input-output tables in dealing with secondary products, in routing and classifying imports of goods and services, and in allocating and classifying trade and transport margins and net indirect taxes.

1. Treatment of secondary products

16. Columns 1 through 6 of table 3, on the treatment of secondary products, are blank in the case of the input-output tables of some countries either because information was not available on the question of how secondary products are treated or because secondary products are not allocated to the activity where they are characteristically produced. Column 4 indicates that in a few of these instances, there is an industry x commodity output (make) matrix, which shows the secondary, as well as characteristic, products of each kind of activity. If secondary products are not negligible and are not shifted to their characteristic kind of producer, the available comprehensive input-output table is a commodity x industry matrix.

17. The information in columns 1 through 3 indicates that the common way of treating secondary products in compiling comprehensive input-output tables is to shift these products and their associated inputs - intermediate and direct - from the industrial activity where they are actually made to the industrial activity where they are characteristically made. The resulting input-output tables are either

industry x industry or commodity x commodity matrices. These shifts are based, most frequently on specially gathered data. Use is also made for this purpose of assumptions of an industry technology (the cost-structure of secondary output is considered to be that of the industrial activity where they are actually produced) or a commodity technology (the cost-structure of secondary output is that of the industrial activity where they are characteristically produced).

18. Easier but less desirable alternatives for shifting secondary products are tabulated in columns 5 and 6 of table 3. These approaches consist of imputing sales of secondary products by their producing kind of activity to their characteristic kind of activity or considering the secondary product to be negative inputs into the former and positive inputs into the latter. The gross output and cost of production of the secondary products are duplicated in the output and input of the producing and characteristic kinds of activity in the former treatment but not in the latter treatment. The resulting input-output tables may be a mixture, to some extent, of industry x industry and commodity x industry data. Each of the two alternatives has been used in five of the analysed sets of input-output tables.

2. Treatment of imports

19. Column 7 of table 3 indicates that imports of goods and services are separated into competitive and complementary in the case of 11 of the 64 sets of input-output tables shown. The small proportions of countries which have drawn this distinction may be due to the difficulties of making it.

20. Where the distinction is made, the competitive imports are cross-classified according to kind of user and kind of characteristic producer or commodity either along with domestic products (six countries) or separated from domestic products (five countries). In some of the former instances, the input-output data are also designed to show the composition of the competitive imports. While the independent cross-classification of competitive imports yields more fruitful data, it is more difficult to carry out than cross-classification together with domestic products because of the problems of allocating the imports to intermediate and final users.

21. In the case of the input-output tables where competitive and complementary imports are not distinguished one from the other, all imports are classified separately from domestic products in 38 of the 53 cases. The imports are cross-classified according to kind of user and kind of characteristic producer or commodity in about two-thirds of these instances and according to kind of user only in the remaining instances. In either case difficulties are encountered in allocating imports to users.

3. Treatment of trade and transport margins

22. Columns 11 through 13 of table 3 indicates that the trade and transport margins on commodities are classified according to the various users of these commodities in somewhat over two-thirds of the comprehensive input-output tables analysed, in other words, the trade and transport margins that are part of the purchasers' values paid by the users, are shown separately. The concomitant of this is that the gross outputs of the commodities are valued at producers' values, that is at prices ex the producing establishments. As will be noted in discussing columns 14 through 17 these producers' values are, in a few instances, subdivided into basic values and net commodity taxes or even into factor values and net indirect taxes. Producers' values are also used in valuing the gross output of commodities in the few cases where trade and transport margins on commodities are cross-classified according to user and either kind of characteristic producers of the commodities or the kind of commodities themselves. In somewhat less than one-third of cases presented in table 3, the gross outputs of commodities are valued at purchasers' values; in other words the trade and transport margins on the commodities are classified according to (allocated to) the kind of activity in which the commodities are produced.

23. Some countries have compiled input-output tables in purchasers' values as well as in producers' values because each type of table has its particular uses. However, producers' values are more commonly used than purchasers' values because, among other matters, the inclusion of trade and transport margins in the latter values distorts input-output coefficients and the comparative analysis of users' demands for commodities.

4. Data on net indirect taxes

24. It is clear from column 15 of table 3 that all countries who in input-output tables, show net indirect taxes or indirect taxes minus subsidies as a separate element of value added, classify these items according to the kind of activities of the producers of the value added.

25. Though it is worthwhile for input-output analysis and for other purposes to separate net commodity taxes from other forms of net indirect taxes and to value input-output data at basic values, this was done in only three of the tables analysed. In these instances, the net commodity taxes are classified according to both the

kind of users of the commodities in question and the kind of commodities or their characteristic producers. Seven other countries have found it feasible to subdivide the purchasers' values of commodities into factor values, total net indirect taxes and trade and transport margins. In these instances, total net indirect taxes are classified, or even cross-classified, according to the kind of users and the kind of commodities involved. Cross-classification is desirable, if feasible, as net indirect taxes vary with both kind of commodity and kind of purchaser.

D. Table 4

26. Table 4 indicates that matrices of direct input-output coefficients and matrices of direct and indirect supply of commodities required per unit of final demand are published in the case of more than half of the input-output studies analysed. The former matrices have been compiled directly from the comprehensive input-output tables listed in table 2; the coefficients concern the ratios of intermediate inputs and, in some cases, also ratios of primary inputs to the gross output of the various industries. The latter matrices have been prepared by inverting the comprehensive input-output tables; they show the direct and indirect contribution of each kind of gross output to meeting the final demand for a unit of a given commodity. In a number of instances, the matrices of direct and indirect requirements per unit of final demand have been prepared on the supply of commodities both from domestic production only and from domestic production plus imports.

27. Matrices of direct and indirect requirements for primary inputs into various kinds of activities per unit of their gross output of commodities or per unit of each type of final demand for the commodities, are furnished in less than one-fifth of the input-output studies set out in table 4. These matrices have been derived by multiplying the coefficients of direct and indirect contribution of each kind of activity per unit of supply of given commodities by the direct input-output coefficients of the primary inputs entering into each contribution.

Table 1. National input-output tables compiled and the tables selected for analysis

Country	Years for which input-output tables are compiled	Input-output table selected for analysis: Year	Input-output table selected for analysis: Published in
(1)	(2)	(3)	(4)
1. Algeria	1954	1954	Ministère de l'Algérie, Perspectives Décennales de Développement Economique de l'Algérie, Algiers, March 1958.
2. Argentina	1950; 1953; 1959; 1960	1960	Argentina, Consejo Nacional de Desarollo, Actualización de la Matriz de Insumo Producto del Año 1953 al Año 1960, Publicación Nr. 18, Buenos Aires 1965.
3. Australia	1946/47; 1953/54; 1955/56; 1958/59	1958/59	Commonwealth Bureau of Census and Statistics, Input-Output Tables 1958/59, Canberra 1963.
4. Austria	1955; 1961	1961	Information obtained through direct communication with the Austrian Statistical Office.
5. Belgium	1953; 1958; 1959; 1965 1/	1965	Institut National de Statistique, Tableau Entrées-Sorties de la Belgique pour 1965, Etudes statistiques, nr. 22, 1970
6. Bolivia	1958	1958	Junta Nacional de Planeamiento, "La Matriz de Transacciones Intersectoriales de Bienes Nacionales e Importados", published in Planeamiento, La Paz, March 1961.
7. Brazil	1959	1959	Instituto de Pesquisa Economico-Social Aplicada (Ministerio do Planejamento e Coordenaçao geral), Relações Inter-industriais no Brasil, December 1967. Mimeographed.
8. Bulgaria	1960; 1963	1963	Information obtained through direct communication with the Bulgarian Statistical Office.
9. Canada	1949; 1961	1961	Dominion Bureau of Statistics, The Input-Output Structure of the Canadian Economy, 1961, Vols. 1 and 2, the Minister of Industry, Trade and Commerce, Ottawa, August 1969.
10. Ceylon	1965	1965	S. Narapalasingham, On the Construction and Implementation of a Planning Model for Ceylon, Thesis University of Bristol, December 1970.
11. Costa Rica	1957	1957	La Universidad de Costa Rica, Escuela de Ciencias Económicas y Sociales, "El Desarollo Economico de Costa Rica, Estudio no. 2, Sector Industrial de la Economía Costarricens con un Modelo Global del Sistema Económico", published in Publicaciones de la Universidad de Costa Rica, Serie Economia y Estadística, no. 6, 1959.
12. Cyprus	1954; 1957	1957	A.J. Meyer, S. Vassiliou, The Economy of Cyprus, Harvard University Press, Cambridge 1962.
13. Czechoslovakia	1956-1959; 1962; 1967	1962	Information obtained through direct communication with the Czechoslovakian Statistical Office.
14. Denmark	1930-1946, annually; 1949; 1953; 1958; 1966 2/	1953	Det Statistiske Departement, Nationalregnskabsstatistik 1947-1960, Statistiske Undersøgelser nr. 7, Copenhagen 1962.
15. Egypt	1954; 1959; 1962	1962	S. Hamid and Fayza H. Omar, Input-Output Analysis, a Pilot Example, August 1963, Memo 360, Mimeographed. Gamal E. Eleish, "The Input-Output Model in a Developing Economy: Egypt", published in Structural Interdependence and Economic Development (Proceedings of an International Conference on Input-Output Techniques, Geneva 1961), editor, Tibor Barna, St. Martin's Press Inc., New York 1963, pp. 199-223.
16. Fiji	1966; 1967	1967	E.C. Dommen, An Input-Output Table for Fiji 1967, Commonwealth Secretariat, London 1970.
17. Finland	1956; 1959; 1965	1965	Tillastollinen Päätoimisto (Central Statistical Office), Panos-tuotostutkimos Suomen Talouselämästä Vuodelta 1965 (Input-Output Study for Finland 1965), Tilastotiedotus, Helsinki 1969.
18. France	1951; 1956; 1959; 1955-1966 2/; 1965 1/	1962	Institut National de la Statistique et des Etudes Economiques, "Les Comptes des Biens et Services, Séries 1959-1966", Les Collections de l'INSEE, nr. 43, Comptes et Planification, Séries C, nr. 10, May 1971. (This publication includes as a separate annex "Tableau d'Echanges Interindustriels, Année 1962").
19. Germany, Federal Republic of	1950; 1954; 1958; 1962; 1965 1/; 1966	1966	Reiner Stäglen and Hans Wessels, "Input-Output Tabelle für die Bundesrepublik Deutschland 1966", published in Vierteljahrshefte zur Wirtschaftsforschung, 3-1971, Deutsches Institut für Wirtschaftsforschung, Berlin 1971.
20. Greece	1954; 1958; 1960	1960	A. Koutsoyiannis and A. Ganas, Input-Output Table of the Greek Economy (Year 1960), Center of Planning and Economic Research, Athens 1967.
21. Guyana	1959	1959	A. Kundu, "Inter-Industry Table for the Economy of British Guiana, 1959 and National Accounts 1957-1960" published in Social and Economic Studies, Supplement to Vol. 12, no. 1, Institute of Social and Economic Research, University of the West Indies, Jamaica, March 1963. A. Kundu, "The Economy of British Guyana 1960-1975", published in Social and Economic Studies, Vol. 12, No. 3, September 1963.

Table 1. National input-output tables compiled and the tables selected for analysis (continued)

Country	Years for which input-output tables are compiled	Input-output table selected for analysis: Year	Input-output table selected for analysis: Published in
(1)	(2)	(3)	(4)
22. Hungary	1957; 1959; 1961; 1959-1964; 1965; 1968	1968	Central Statistical Office, Department of Industrial Statistics, Input-Output Table of the Hungarian National Economy, 1970.
23. India	1949/50-1953/54 annually; 1955/61; 1960/61; 1964/65	1964/65	M.R. Saluja, "Structure of the Indian Economy", Sankhya - The Indian Journal of Statistics, Series B, Vol. 30, Parts 1 and 2, June 1968, pp. 97-122.
24. Iran	1961; 1965	1965	Amir Shahpour Shaheen, 1965 Input-Output Table for Iranian Economy, Mimeographed.
25. Iraq	1960-1963, annually	1963	Taber Hamdi Kanaan, Input-Output and Social Accounts of Iraq 1960-1963, Ministry of Planning, Baghdad, September 1965.
26. Ireland	1956; 1960; 1964	1964	Central Statistical Office, Input-Output Tables for 1964, Dublin, January 1970.
27. Israel	1958; 1968/69	1968/69	David Chen, Input-Output Tables 1968/69, Israel Central Bureau of Statistics, Special Series No. 380, Jerusalem 1972.
28. Italy	1950; 1953; 1959; 1965;[1] 1967	1967	Istituto Centrale di Statistica, "Tavola Intersettoriale dell'Economia Italiana per l'Anno 1967", published in Supplemento Straordinario al Bolletino Mensile di Statistica, nr. 11, November 1970.
29. Ivory Coast	1960	1960	H. Leroux, J.P. Allier, Planification en Afrique, Tome VI, Fonctione de Production et Modèles, République Française, Ministère de la Coopération, Paris, 1964, pp. 122, 125-129.
30. Jamaica	1958	1958	Carlun O'Loughlin, "Long-Term Growth of the Economy of Jamaica", Social and Economic Studies, Vol. 12, No. 3, University of the West Indies, Jamaica, September 1963, pp. 246-282.
31. Japan	1951; 1955; 1960; 1963; 1965	1965	Bureau of Statistical Standards, in cooperation with other Government Agencies, Input-Output Table for 1965, published in 1969 (in Japanese).
32. Jordan	1968	1968	Department of Statistics, The National Accounts and Input-Output Analysis 1967-1968, Amman, December 1969.
33. Korea, Republic of	1960; 1966; 1968	1968	Bank of Korea, Input-Output Table 1968, 1970.
34. Lebanon	1964	1964	Ministère du Plan, Direction Centrale de la Statistique, Les Comptes Economiques de l'Année 1964, Volume 1, Resultats, Beyrouth, May 1967
35. Malawi	1968	1968	Information obtained through direct communication with Statistical Office in Malawi.
36. Malaysia	1960	1960	United Nations, Economic Survey of Asia and the Far East 1964, New York 1965, pp. 28-39.
37. Mali	1958	1958	H. Leroux, J.P. Allier, Planification en Afrique, Tome VI, Fonctions de Production et Modèlee, République Française, Ministère de la Coopération, Parie 1964, pp. 119-124.
38. Malta	1961-1968, annually	1968	Central Office of Statistics, National Accounts of the Maltese Islands, Malta, 1969.
39. Mexico	1950; 1960	1960	Banco de Mexico S.A., Cuadro de Insumo Producto de Mexico 1960, Mexico, December 1966.
40. Morocco	1958	1958	Ministère de l'Economie et des Finances, Plan quinquennal 1960-1964, le Développement Industrial, 1961.
41. Netherlands	1938; 1946; 1948-1957, annually; 1958-1960, annually; 1961-1964, annually; 1965[1]	1964	Centraal Bureau voor de Statistiek, De Produktiestruktuur van de Nederlandse Volkshuishouding, Deel IV. Input-Output Tabellen 1961-1964, The Hague 1968.
42. New Zealand	1952/53; 1954/55; 1959/60	1959/60	Department of Statistics, Interindustry Study of the New Zealand Economy 1959/60, parts 1-4, Wellington, 1966 and 1967.
43. Nigeria	1959/60	1959/60	Nicholas G. Carter, An Input-Output Analysis of the Nigerian Economy, Massachusette Institute of Technology, Cambridge, 2nd printing, March 1966.
44. Norway	1948; 1950; 1954; 1959; 1964	1964	Statistisk Sentralbyrå, Input-Output Data, 1954, 1959 and 1964, Oslo, 1968.
45. Pakistan	1954; 1955; 1959/60; 1960/61	1960/61	Gulam Rasul, A Summary of Input-Output Studies of the Economy of Pakistan, Research Report No. 33, Pakistan Institute of Development Economics, Karachi, July 1965.

Table 1. National input-output tables compiled and the tables selected for analysis (continued)

Country	Years for which input-output tables are compiled	Input-output table selected for analysis: Year	Published in
(1)	(2)	(3)	(4)
46. Peru	1950; 1955; 1956; 1957; 1959	1959	Banco Central de Reserva del Perú, Renta Nacional del Perú 1942-1966, 1962.
47. Philippines	1956; 1961	1961	Office of Statistical Coordination and Standards, The 1961 Interindustry (Input-Output) Accounts of the Philippines, Manila, 1969.
48. Poland	1951; 1958-1962, annually; 1967[2/]	1962	Glówny Urzad Statystyczny, Bilans Przepływów Międzygałęziowych w Gospodarce Narodowej Polski - 1962, Warsaw, 1966.
49. Portugal	1957; 1959	1959	J. Silva Ferreira, Algumas Aplicações da Análise Input-Output à Matriz Portuguesa de 1959, published by Instituto Gulbenkian de Ciência, Centro de Economia e Finanças, Lisbon, 1967.
50. Senegal	1959	1959	H. Leroux, J.P. Allier, Planification en Afrique; Tome VI, Fonctions de Production et Modèles, Republique Française, Ministère de la Coopération, Paris 1964, pp. 147-170.
51. South Africa	1951/52; 1956/57; 1959/60[2/]	1956/57	D.C. Krogh, "An Input-Output Analysis of the South African Economy 1956/57", The South African Journal of Economics, Vol. 29, 1961, pp. 258-273.
52. Spain	1954-1958, annually; 1962	1962	Organizacion Sindical Española, Tabla Input-Output de la Economia Española, Vols. 1 and 2, Pueblo, Madrid, 1962.
53. Sudan	1955/56; 1961/62; 1962/63[2/]	1961/62	Department of Statistics, National Income of Sudan in 1961/62 (with preliminary estimates for 1962/63), Khartoum, May 1964.
54. Sweden	1957; 1961; 1964	1964	Statistiska Centralbyrån, Input-Outputtabeller för Sverige 1964, Statistiska Meddelanden N 1970: 13, Stockholm, April 1970.
55. Togo	1956; 1957; 1958	1958	Banque Centrale des Etats de l'Afrique de l'Ouest, Comptes Economiques Togo 1956-1957-1958, Etudes Economiques Ouest Africaines No. 3.
56. Trinidad and Tobago	1959; 1962	1962	A.A. Francis, "A note on Interindustry Relations in the Economy of Trinidad and Tobago, 1962", published in Research Papers No. 2, Central Statistical Office, December 1965.
57. Tunisia	1957	1957	United Nations, Perspectives de Développement Economique en Tunisie 1957-1971 et Comptes Economiques et Tableaux Input-Output pour 1957, United Nations Technical Assistance Programme, 1961, Report RAO/TUN/3.
58. Turkey	1959; 1963	1963	A.K. Chakraverti; C.Cinar, G. Canalp, Structural Interdependence of the Turkish Economy 1963, State Planning Organization, Istanbul 1970.
59. Union of Soviet Socialist Republics	1923/24; 1959; 1966	1966	Central Statistical Administration, Statistical Yearbook 1967 (in Russian). Vladimir G. Trel, "Characteristics of the USSR Input-Output Tables", published in International Comparisons of Inter-industry Data, Industrial Planning and Programming Series No. 2, United Nations, New York 1969, pp. 133-153. M.R. Eydelman, Input-Output Table of Social Product: USSR, U.S. Department of Commerce, Clearing House for Federal Scientific and Technical Information, Washington, D.C., November 1966.
60. United Kingdom	1935; 1948; 1954; 1960; 1963	1963	Central Statistical Office, Input-Output Tables for the United Kingdom 1963, Studied in Official Statistics No. 16, London, 1970.
61. United States	1919; 1929; 1939; 1947; 1958; 1963; 1966[2/]	1963	U.S. Department of Commerce, Office of Business Economics, "Input-Output Structure of the U.S. Economy, Vols. 1-3", Supplement to the Survey of Current Business, 1969.
62. Yugoslavia	1955; 1958; 1962; 1964; 1966	1966	Savezni Zavod za Statistiku, Medusobni Odnosi Privednih Delatnosti Yugoslavije u 1966 (Interindustry Relations of the Yugoslav Economy in 1966), Series "Studije, Analize i Prikazi 42", Belgrade 1969. Savezni Zavod za Statistiku, Medusobni Odnosi Privednih Delatnosti Yugoslavije 1962 i 1966, Uvoz po Delatnostima Porekla i Namene (Imports by Industries of Origin and Destination), Series "Studije, Analize Prikazi 50", Belgrade 1970.
63. Zaïre	1957	1957	Carbonnelle, C., - Kirschen, E.S., Structure de l'Economie de Haut et Bas Uêle en 1957, Mimeographed, May 1960.
64. Zambia	1961; 1964; 1965; 1967	1967	Central Statistical Office, National Accounts 1964-1967 and Input-Output Table 1967, Lusaka 1970.

See the following page for footnotes to table 1.

Footnotes to table 1

1/ Belgium, France, Rederal Republic of Germany, Italy and the Netherlands. The input-output tables relating to 1965 for these countries which were compiled in accordance with the standardized form of the Statistical Office of the European Economic Community have not been selected for analysis. The tables analysed have been restricted to nationally designed tables. In the case of Belgium, this input-output table relates to the same year - 1965 - as the one prepared for the European Economic Community.

2/ Denmark, France, Poland, South Africa, Sudan and the United States. Because in the case of these countries, detailed information was not available on the latest input-output table, a table for an earlier year has been analysed. In the case of France, the 1962 table has been analysed because it is the basic table from which the annual input-output tables for the period 1959-1966 were derived.

Table 2. Classes in the input-output table, categories of final demand and categories of primary input

Country	Year to which analysed input-output table refers	Basic tables available		Ind. and comm. categories				Categories of final demand									Categories of primary input										
				Number of classes		Classification											Compensation of employees				Indirect taxes minus subsidies						
		Com-pre-hen-sive input matrix	Supple-mentary ind. x comm. output matrix	Comm. rows	Ind. col-umns	Nation-al	Other	Sum of consump-tion and gross fixed capital forma-tion	Sum of public and private consump-tion	Sum of public consump-tion and public gross fixed capital forma-tion	Pri-vate con-sump-tion	Pub-lic con-sump-tion	Pri-vate gross fixed capi-tal forma-tion	Public gross fixed capi-tal forma-tion	Change in stocks	Ex-ports	Total	Wages and salaries	Employers' contri-bution to social security schemes	Depre-ciation	Comm. taxes	Import duties	Minus sub-sidies	Rest of indirect taxes minus subsidies	Opera-ting surplus	Rest of value added	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
1. Algeria	1954	x	x	25 1,2/	25	x		x							x	x		x	x	x			x	x	x 38/		1.
2. Argentina	1960	x		28	28	x					x	x		x	x	x										x	2.
3. Australia	1958/59	x		34	34	x				x	x		x		x	x 29/	x					x		x		x	3.
4. Austria	1961	x		40	40	x					x	x	x	x	x	x 30/		x	x	x			x	x	x		4.
5. Belgium	1965	x	x	115	81	x	NIMEXE NACE				x	x		x	x	x 30/		x	x	x				x	x		5.
6. Bolivia	1958	x		10	10	x					x	x		x	x	x										x	6.
7. Brazil	1959	x		32	32	x					x	x		x		x	x									x	7.
8. Bulgaria	1963	x		19	19	x			x 7/					x 7/	x 7/	x 7/		x 7/						x 7/	x 7/		8.
9. Canada	1961	x	x	644 3/	187 3/	x				x	x		x		x	x	x				x		x	x	x 39/		9.
10. Ceylon	1965	x		41	41		ISIC				x	x		x	x	x								x		x	10.
11. Costa Rica	1957	x		19 4/	17 4/	x			x 8/					x 8/		x						x				x	11.
12. Cyprus	1957	x		61	61	x					x	x		x	x	x										x	12.
13. Czechoslovakia	1962	x		96	96	x					x 7/	x 7/		x 7/	x 7/	x 7/		x 7/		x 7/				x 7/	x 7/		13.
14. Denmark	1953	x	x	19	19		SITC		x					x 20/	x	x	x					x		x	x		14.
15. Egypt	1962	x		13	13	x					x	x	x	x	x	x	x							x		x	15.
16. Fiji	1967	x	x	4 5/	15 5/	x					x 9/	x		x 21/		x 31/	x					x 34/	x	x		x 42/	16.
17. Finland	1965	x	x	66	66	x	ISIC				x	x		x	x	x	x							x		x	17.
18. France	1962	x	x	78 1/	78	x					x 10/	x	x 22/	x	x 27/	x						x				x	18.
19. Germany, Federal Republic of	1966	x		54	54	x					x 11/	x 11/		x	x	x	x			x				x	x		19.
20. Greece	1960	x		50	50	x					x	x		x	x	x	x							x		x	20.
21. Guyana	1959	x		14	14	x					x	x		x		x								x		x	21.

Table 2. Classes in the input-output table, categories of final demand and categories of primary input (continued)

Country	Year to which analysed input-output table refers	Basic tables available: Comprehensive input matrix	Basic tables available: Supplementary ind. x comm. output matrix	Ind. and comm. categories: Number of classes: Comm. rows	Ind. and comm. categories: Number of classes: Ind. columns	Ind. and comm. categories: Classification: National	Ind. and comm. categories: Classification: Other	Categories of final demand: Sum of consumption and gross fixed capital formation	Sum of public and private consumption	Sum of public consumption and public gross fixed capital formation	Private consumption	Public consumption	Private gross fixed capital formation	Public gross fixed capital formation	Change in stocks	Exports	Categories of primary input: Compensation of employees: Total	Wages and salaries	Employers' contribution to social security schemes	Depreciation	Indirect taxes minus subsidies: Comm. taxes	Import duties	Minus subsidies	Rest of indirect taxes minus subsidies	Operating surplus	Rest of value added	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
22. Hungary	1968	x		83	83	x			x7/					x7/	x7/	x7/		x7/								x7/	22.
23. India	1964/65	x		77	77	x					x	x		x	x	x				x						x	23.
24. Iran	1965	x		30	30		ISIC				x	x		x20/		x	x			x				x	x		24.
25. Iraq	1963	x		32	32		ISIC				x	x		x	x	x	x							x		x	25.
26. Ireland	1964	x	x	92	92	x					x	x12/		x	x	x29/	x			x	x35/		x	x35/	x		26.
27. Israel	1968/69	x		30	30	x					x	x		x	x	x	x					x36/	x36/	x		x	27.
28. Italy	1967	x		77	77	x			x					x	x	x		x	x	x		x		x	x		28.
29. Ivory Coast	1960	x	x	21 1/	21						x13/	x	x23/	x	x	x	x							x		x	29.
30. Jamaica	1958	x		17	17	x				x	x			x		x	x							x		x	30.
31. Japan	1965	x	x	156	156	x					x14/	x		x	x	x	x			x				x	x		31.
32. Jordan	1968	x		39	39	x					x	x		x	x	x	x					x				x43/	32.
33. Korea, Republic of	1968	x		43	43	x					x14/	x		x	x	x										x	33.
34. Lebanon	1964	x		14 12/	14	x					x	x	x	x	x	x										x	34.
35. Malawi	1968	x		10	10	x			x					x	x	x	x							x		x	35.
36. Malaysia	1960	x		34	34	x			x					x	x	x								x		x	36.
37. Mali	1958	x		8 1/	8	x			x					x		x										x	37.
38. Malta	1968	x		24	24	x				x	x			x		x	x							x		x	38.
39. Mexico	1960	x		45	45	x					x	x	x	x	x	x	x			x				x	x		39.
40. Morocco	1958	x		31 1/	31	x					x	x		x24/	x	x	x						x	x		x	40.
41. Netherlands	1964	x		35	35	x	ISIC				x	x	x	x	x	x		x	x	x			x	x	x		41.
42. New Zealand	1959/60	x		110	110	x	ISIC				x	x		x	x	x	x			x				x	x		42.
43. Nigeria	1959/60	x		20	20	x			x					x		x										x	43.
44. Norway	1964	x	x	36 2/	36	x					x	x		x	x	x	x			x				x	x		44.
45. Pakistan	1960/61	x		30	30	x	ISIC	x6/																x		x	45.

Table 2. Classes in the input-output table, categories of final demand and categories of primary input (continued)

Country	Year to which analysed input-output table refers	Basic tables available: Com-pre-hen-sive input matrix	Basic tables available: Supple-mentary ind. x comm. output matrix	Ind. and comm. categories: Number of classes: Comm.	Ind. and comm. categories: Number of classes: Ind.	Ind. and comm. categories: Classification: Nation-al	Ind. and comm. categories: Classification: Other	Categories of final demand: Sum of consump-tion and gross fixed capital forma-tion	Sum of public and private consump-tion	Sum of public consump-tion and public gross fixed capital forma-tion	Pri-vate con-sump-tion	Pub-lic con-sump-tion	Pri-vate gross fixed capi-tal forma-tion	Public gross fixed capi-tal forma-tion	Change in stocks	Ex-ports	Categories of primary input: Compensation of employees: Total	Compensation of employees: Wages and salaries	Compensation of employees: Employers' contri-bution to social security schemes	Depre-ciation	Indirect taxes minus subsidies: Comm. taxes	Indirect taxes minus subsidies: Import duties	Indirect taxes minus subsidies: Minus sub-sidies	Indirect taxes minus subsidies: Rest of indirect taxes minus subsidies	Opera-ting surplus	Rest of value added	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
46. Peru	1959	x		8	8	x		x							x	x	x			x				x	x 40/		46.
47. Philippines	1961	x		198	198	x	ISIC				x	x		x	x	x	x			x				x	x		47.
48. Poland	1962	x		144	144	x					x 7/	x 7/		x 7/	x 7/	x 7/		x 7/						x 7/	x 7/41/		48.
49. Portugal	1959	x		18	18	x					x	x		x	x	x										x	49.
50. Senegal	1959	x		18	18	x					x 15/	x		x	x	x	x 32/					x		x		x	50.
51. South Africa	1956/57	x		48	48		ISIC				x	x		x	x	x	x							x		x	51.
52. Spain	1962	x		86	86	x					x	x	x	x	x	x		x	x	x				x 37/	x		52.
53. Sudan	1961/62	x		14	14	x					x	x 16/		x	x	x								x		x	53.
54. Sweden	1964	x	x	32	32	x	(SITC (ISIC				x	x 12/		x	x	x	x			x				x	x		54.
55. Togo	1958	x	x	8 2/	10	x					x	x	x	x 25/	x	x		x	x				x	x		x	55.
56. Trinidad and Tobago	1962	x		13	13		ISIC				x	x		x	x	x	x			x				x	x		56.
57. Tunisia	1957	x		21	21	x			x					x	x	x		x	x	x			x	x	x		57.
58. Turkey	1963	x		37	37	x	ISIC				x	x	x	x	x	x								x		x	58.
59. Union of Soviet Socialist Republics	1966	x		110	110	x					x 7/	x 7/18/		x 7/26/	x 7/	x 7/		x 7/33/	x					x 7/	x 7/44/		59.
60. United Kingdom	1963	x	x	73	73	x					x	x		x	x	x	x							x		x	60.
61. United States	1963	x	x	367	367	x	ISIC			x	x		x		x	x										x	61.
62. Yugoslavia	1966	x		98	98	x					x	x		x	x 28/	x	x			x						x	62.
63. Zaïre	1957	x		14	14	x					x 19/	x		x	x	x	x 32/			x						x	63.
64. Zambia	1967	x		38	38		ISIC				x	x		x	x	x	x			x	x		x	x	x		64.

See the following pages for general note and footnotes to table 2.

General note and footnotes to table 2

1. General note

The acrononyms for the international classifications listed in column 6 are as follows: ISIC for the International Standard Industrial Classification; NACE and NIMEXE for the standard classifications of economic activities and of external trade, respectively, of the Statistical Office of the European Economic Community; and SITC for the Standard International Trade Classification.

2. Specific footnotes

Columns 1 through 6, Industry and commodity categories

1/ Algeria, France, Ivory Coast, Lebanon, Mali and Morocco. The outputs of industries are valued at purchasers' values and trade and transport margins are presented in a row separately from the rows for intermediate consumption to the input-output tables but not in a commodity row. The number of rows in the inner input-output table is therefore one less than the number of columns. The separate row for trade and transport margins is classified as follows in the case of each country: Algeria, into margins on domestic trade, imports and exports; France, into margins on intermediate consumption, household consumption, production for own consumption, government consumption, gross fixed capital formation and exports; Ivory Coast, Lebanon and Mali, into trade and transport margins; Morocco, into margins on local goods and imported goods.

2/ Algeria, Lebanon, Norway and Togo. The categories of commodities used in classifying the output data are not identical with those used in classifying the input data. The categories of commodities used in classifying the input (absorption) data but not those used in classifying the output (make) data match the categories of industries used for these purposes. For example, in the case of Algeria, the categories of commodities used in classifying the output data are commodities for intermediate consumption subdivided into agricultural goods, other basic materials, energy, other goods for intermediate consumption and services for intermediate consumption and commodities for final use subdivided into foodstuffs, other goods for final consumption, services for final consumption and equipment while the categories of commodities used in classifying the input data match the categories of industries used for this purpose.

3/ Canada. Tables with a classification of 644 categories of commodities and 187 categories of industries are available but are not published. The largest published tables contain 197 categories of commodities and 110 categories of industries.

4/ Costa Rica. The table is incomplete as the outputs but not the inputs of agriculture and mining are covered.

5/ Fiji. The input-output table is rectangular; only four categories of intermediate inputs are distinguished, namely, repair and maintenance, electricity and water, other locally produced inputs and transport and distribution charges.

Columns 7 through 15, Categories of final demand

The footnotes below concern differences in the scope and detail of classification of final demand between the countries in question and the columns of the table.

6/ Pakistan. Only total final demand is shown.

7/ Bulgaria, Czechoslovakia, Hungary, Poland and USSR. The services, wage and salary payments and gross profits of the non-material sphere of the economy are not covered as such in the input-output tables of these countries. The goods and material services consumed in the non-material sphere of the economy are included in private or public consumption, as appropriate. A form of final disposition of goods not shown in table 2 is losses in stocks, fixed assets, etc. Depreciation is classed as part of intermediate consumption not as part of primary inputs.

8/ Costa Rica. The sum of public and private consumption is subdivided into durable and non-durable goods. Gross fixed capital formation in the form of construction projects covers expenditures on construction materials only.

9/ Fiji. Private consumption is subdivided into tourists and other.

10/ France. Private consumption includes the costs of the services of financial institutions which are considered themselves to consume these services. The consumption expenditure all related to financial institutions is distinguished from that of households.

11/ Federal Republic of Germany. Private consumption is classified into that of private households and that of private non-profit organizations. Public consumption is subdivided for non-military and military purposes.

12/ Ireland and Sweden. Public consumption is subdivided into expenditures by central government and local government.

13/ Ivory Coast. Private consumption includes the costs of the services of financial institutions, which are considered themselves to consume these services, and is subdivided into the outlays of the households and the outlays of financial institutions.

14/ Japan and Republic of Korea. Private consumption includes certain outlays of businesses and government, for example expenditures on employees' travel and entertainment in connexion with their work.

15/ Senegal. Private consumption is classified into that of African and European households.

16/ Sudan. Included are current expenditures in respect of the educational services of non-governmental organizations.

18/ USSR. Public consumption is classified according to purpose.

19/ Zaïre. Private consumption is classified into that of the household of nationals and of foreigners.

20/ Denmark and Iran. Gross fixed capital formation is classified according to the kind of economic activity of users.

21/ Fiji. Gross fixed capital formation is classified into building and works and other according to kind of economic activity of users.

22/ France. Private gross fixed capital formation is classified into that of non-financial enterprises and financial institutions.

23/ Ivory Coast. Private gross fixed capital formation is subdivided into that of non-financial enterprises and financial institutions.

24/ Morocco. Gross fixed capital formation is divided into own account construction and other.

25/ Togo. Public gross fixed capital formation is classified according to national administrations, foreign administrations and development funds.

26/ USSR. Gross fixed capital formation is classified into: for expansion of stocks of fixed as ets, for replacement of worn-out fixed assets and for replacement of destroyed fixed and circulating assets.

27/ France. Changes in stocks are subdivided into products and intermediate materials and supplies.

28/ Yugoslavia. Increases and decreases in stocks are shown separately.

29/ Australia and Ireland. Exports are divided into goods and services.

30/ Austria and Belgium. Exports are classified according to region.

31/ Fiji. Exports and re-exports are distinguished.

Columns 16 through 25, Categories of primary inputs

The footnotes below describe differences in scope and detail of classification between the countries in question and the columns of the table on primary inputs.

32/ Senegal and Zaïre. Distinguished are wages and salaries paid to nationals and to foreigners.

33/ USSR. Distinguished are wages and salaries paid by collective farms and by other enterprises.

34/ Fiji. Import duties on imports for intermediate consumption and for final use are presented separately.

35/ Ireland. Indirect taxes are subdivided into nine categories.

36/ Israel. Subsidies are subdivided into subsidies on domestic production and on imports.

37/ Spain. Distinguished are indirect taxes on producers and other indirect taxes.

38/ Algeria. Operating surplus is subdivided into profits after taxes and direct taxes.

39/ Canada. The income of unincorporated business is distinguished from the operating surplus of other enterprises.

40/ Peru. Operating surplus is classified into rents and profits.

41/ Poland. Net income is classified into that of state-owned, co-operative and private enterprises.

42/ Fiji. Gross profits, inclusive of depreciation, are subdivided into that of government enterprises, of private corporate enterprises and of unincorporated enterprises.

43/ Jordan. Value added reduced by compensation of employees and import duties is subdivided into rent, interest and remaining income of own account workers and of other enterprises.

44/ USSR. Value added of the material sphere reduced by wages and salaries is classified into that of collective farms, of state enterprises and of private plots.

Table 3. Treatment of selected flows

Country	Year to which analysed input-output table refers	Secondary products: Transfer of outputs and corresponding inputs to characteristic industry: Based on gathering special additional information	Secondary products: Transfer of outputs and corresponding inputs to characteristic industry: Based on assumption of industry technology	Secondary products: Transfer of outputs and corresponding inputs to characteristic industry: Based on assumption of commodity technology	Secondary products: Other treatments: Shown in industry x commodity matrix	Secondary products: Other treatments: As sold to characteristic industry	Secondary products: Other treatments: In separate row, as negative input prod. ind., positive input charact. ind.	Imports: Competitive and complementary imports distinguished	Imports: Classification of all imports or of competitive imports only, if distinguished: According to domestic users only	Imports: Classification of all imports or of competitive imports only, if distinguished: Together with domestic products	Imports: Classification of all imports or of competitive imports only, if distinguished: Separated from domestic products	Trade and transport margins: Classified according to commodities involved	Trade and transport margins: Classified according to users of the commodities	Trade and transport margins: Cross-classified according to commodities involved and users	Indirect taxes minus subsidies (excluding import duties): Net commodity taxes and other net indirect taxes distinguished	Indirect taxes minus subsidies (excluding import duties): Classification of all net domestic indirect taxes or of net domestic commodity taxes if distinguished: According to producers of commodities	Indirect taxes minus subsidies (excluding import duties): Classification of all net domestic indirect taxes or of net domestic commodity taxes if distinguished: According to users of commodities	Indirect taxes minus subsidies (excluding import duties): Classification of all net domestic indirect taxes or of net domestic commodity taxes if distinguished: According to commodities involved or producers and users of the commodities	Statistical discrepancies: Included as "dummy" industry	Statistical discrepancies: Other forms of presentation	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
1. Algeria	1954				x					x		x 14/				x					1.
2. Argentina	1960					x					x	x				21/				x	2.
3. Australia	1958/59	x 1/				x		x			x	x	x			x				x	3.
4. Austria	1961									x			x			x					4.
5. Belgium	1965						x	x			x		x			x					5.
6. Bolivia	1958	x									x		x			21/					6.
7. Brazil	1959			x						x			x			21/			x		7.
8. Bulgaria	1963	x								x			x						x		8.
9. Canada	1961		x		x			x		x			x		x	x	x				9.
10. Ceylon	1965	x						x		x				x 16/				x 16/			10.
11. Costa Rica	1957										x	x				21/				x	11.
12. Cyprus	1957										x		x			21/				x	12.
13. Czechoslovakia	1962	x 2/												x		21/					13.
14. Denmark	1953			x 7/	x 7/						x 10/		x			x					14.
15. Egypt	1962										x		x								15.
16. Fiji	1967									x		x				x					16.
17. Finland	1965		x		x						x		x				x				17.
18. France	1962						x			x		x 14/				21/					18.
19. Germany, Federal Republic of	1966								x				x			x					19.
20. Greece	1960					x					x	x				x					20.
21. Guyana	1959								x			x				x					21.

Table 3. Treatment of selected flows (continued)

Country	Year to which analysed input-output table refers	Secondary products: Transfer of outputs and corresponding inputs to characteristic industry: Based on gathering special additional information	Secondary products: Transfer...: Based on assumption of industry technology	Secondary products: Transfer...: Based on assumption of commodity technology	Secondary products: Other treatments: Shown in industry x commodity matrix	Secondary products: Other treatments: As sold to characteristic industry	Secondary products: Other treatments: In separate row, as negative input prod. ind., positive input char-act. ind.	Imports: Competitive and complementary imports distinguished	Imports: Classification of all imports or of competitive imports only, if distinguished: According to domestic users only	Imports: Classification...: Together with domestic products	Imports: Classification...: Separated from domestic products	Trade and transport margins: Classified according to commodities involved	Trade and transport margins: Classified according to users of the commodities	Trade and transport margins: Cross-classified according to commodities involved and users	Indirect taxes minus subsidies (excluding import duties): Net commodity taxes and other net indirect taxes distinguished	Indirect taxes minus subsidies: Classification of all net domestic indirect taxes or of net domestic commodity taxes if distinguished: According to producers of commodities	Indirect taxes...: Classification...: According to users of commodities	Indirect taxes...: Classification...: According to commodities involved or producers and users of the commodities	Statistical discrepancies: Included as "dummy" industry	Statistical discrepancies: Other forms of presentation	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
22. Hungary	1968	x									x		x			21/					22.
23. India	1964/65									x			x			21/				x	23.
24. Iran	1965										x		x			x					24.
25. Iraq	1963					x 8/		x		x			x 17/			x			x	x	25.
26. Ireland	1964				x		x 9/	x		x			x			x	x				26.
27. Israel	1968/69						x 9/				x		x		x	x	x				27.
28. Italy	1967	x									x	x	x			x					28.
29. Ivory Coast	1960									x		x 14/				x					29.
30. Jamaica	1958								x				x								30.
31. Japan	1965	x 3/					x				x			x 18/		x				x	31.
32. Jordan	1968								x				x			21/					32.
33. Korea, Republic of	1968	x						x			x 11/		x			21/				x	33.
34. Lebanon	1964				x						x	x 14/				21/				x	34.
35. Malawi	1968								x				x			x					35.
36. Malaysia	1960								x				x			x				x	36.
37. Mali	1958									x		x 14/				x					37.
38. Malta	1968								x				x			x					38.
39. Mexico	1960								x				x			x					39.
40. Morocco	1958									x		x 14/				x					40.
41. Netherlands	1964							x			x		x					x		x	41.
42. New Zealand	1959/60							x			x 12/		x		x	x					42.
43. Nigeria	1959/60								x				x			21/					43.
44. Norway	1964	x			x						x 13/			x		x			x	x	44.
45. Pakistan	1960/61								x				x			x				x	45.

Table 3. Treatment of selected flows (continued)

Country	Year to which analysed input-output table refers	Secondary products: Transfer of outputs and corresponding inputs to characteristic industry: Based on gathering special additional information	Secondary products: Transfer of outputs and corresponding inputs to characteristic industry: Based on assumption of industry technology	Secondary products: Transfer of outputs and corresponding inputs to characteristic industry: Based on assumption of commodity technology	Secondary products: Other treatments: Shown in industry x commodity matrix	Secondary products: Other treatments: As sold to characteristic industry	Secondary products: Other treatments: In separate row, as negative input prod. ind., positive input charact. ind.	Imports: Competitive and complementary imports distinguished	Imports: Classification of all imports or of competitive imports only, if distinguished: According to domestic users only	Imports: Classification of all imports or of competitive imports only, if distinguished: Together with domestic products	Imports: Classification of all imports or of competitive imports only, if distinguished: Separated from domestic products	Trade and transport margins: Classified according to commodities involved	Trade and transport margins: Classified according to users of the commodities	Trade and transport margins: Cross-classified according to commodities involved and users	Indirect taxes minus subsidies (excluding import duties): Net commodity taxes and other net indirect taxes distinguished	Indirect taxes minus subsidies (excluding import duties): Classification of all net domestic indirect taxes or of net domestic commodity taxes if distinguished: According to producers of commodities	Indirect taxes minus subsidies (excluding import duties): Classification of all net domestic indirect taxes or of net domestic commodity taxes if distinguished: According to users of commodities	Indirect taxes minus subsidies (excluding import duties): Classification of all net domestic indirect taxes or of net domestic commodity taxes if distinguished: According to commodities involved or producers and users of the commodities	Statistical discrepancies: Included as "dummy" industry	Statistical discrepancies: Other forms of presentation	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
46. Peru	1959								x				x			x				x	46.
47. Philippines	1961									x				x		x				x	47.
48. Poland	1962										x			x		x				x	48.
49. Portugal	1959									x						21/					49.
50. Senegal	1959										x		x			x					50.
51. South Africa	1956/57										x10/	x	x			x					51.
52. Spain	1962										x		x			x					52.
53. Sudan	1961/62								x				x19/			x19/	x19/				53.
54. Sweden	1964		x		x			x		x		x	x					x22/		x	54.
55. Togo	1958				x					x		x				x				x	55.
56. Trinidad and Tobago	1962								x			x				x					56.
57. Tunisia	1957										x	x	x			x					57.
58. Turkey	1963									x				x		x					58.
59. Union of Soviet Socialist Republics	1966	x4/	x4/							x		x				x					59.
60. United Kingdom	1963		x5/	x5/	x						x		x			x	x				60.
61. United States	1963					x		x		x			x			21/			x		61.
62. Yugoslavia	1966	x									x		x20/			21/					62.
63. Zaïre	1957								x				x			21/					63.
64. Zambia	1967										x		x			x				x	64.

See the following pages for footnotes to table 3.

Footnotes to table 3

Columns 1 through 6, Secondary products

1/ Australia. The transfers of inputs consist of only intermediate inputs which are clearly associated with the transferred secondary products. Intermediate inputs such as repair and maintenance, power, fuel and light and primary inputs are not transferred.

2/ Czechoslovakia. Transfers of secondary products and associated inputs are limited to manufacturing industries; trnasfers are not made in the case of such industries as agriculture, forestry, fishing and construction.

3/ Japan. The data sought on inputs deal with the inputs into each product excepting by-products and scrap.

4/ USSR. Transfers in the case of the input-output table in natural units are based on the collection of special information from industries; transfers in the case of the input-output table in monetary values are, in general, dealt with on the assumption of an industry technology.

5/ United Kingdom. The assumption of a commodity technology is used in transferring the outputs and inputs of subsidiary secondary products; the assumption of an industry technology issued in transferring the inputs and outputs of by-products.

7/ Denmark. Separate industry x commodity output data are not actually published. However, in compiling the published input-output table, use is made of a classification of output according to kind of industry of 1600 commodity classes.

8/ Iraq. The secondary outputs are transferred as positive inputs to their characteristic industry but are presented separately from the ordinary inputs.

9/ Ireland and Israel. Inputs equivalent to transferred secondary products are not recorded in a separate row. In the case of Ireland, they are treated as negative inputs into the producing industry from the characteristic industry counterbalanced by positive inputs into the producing industry from the producing industry. In the case of Israel, they are treated as positive inputs into the characteristic industry from the producing industry counterbalanced by negative inputs into the characteristic industry from the characteristic industry.

Columns 7 through 10, Imports

10/ Denmark and South Africa. Separate cross-classifications of all imports according to user and characteristic producer are available in basic worksheets but are not published.

11/ Republic of Korea. The cross-classification of imports is given separately for competitive and complementary imports.

12/ New Zealand. The data on the cross-classification of imports are estimated by assigning the ratios of domestically produced commodities and competing imports for all commodities of a given class to each user of these commodities.

13/ Norway. All imports are considered to be complementary.

Columns 11 through 13, Trade and transport margins

14/ Algeria, France, Ivory Coast, Lebanon, Mali and Morocco. There is in the case of some of these countries also a limited amount of cross-classification according to the use of the commodities and their industrial origin.

16/ Ceylon. The cross-classifications of trade and transport margins is in terms of percentage mark-up only. This is also the case in the cross-classification of indirect taxes minus subsidies.

17/ Iraq. All transactions, except those in crude oil and petroleum products, are valued at producers' values.

18/ Japan. The cross-classification of trade and transport margins is presented in a separate table.

19/ Sudan. The output of agriculture and other industries, excepting manufacturing, is valued at basic values. The output of manufacturing is valued at purchasers' values.

20/ Yugoslavia. Indirect taxes other than import duties are allocated to the distributive trades. Thus trade margins include these taxes.

Columns 14 through 17, Indirect taxes minus subsidies (excluding import duties)

21/ Argentina, Bolivia, Brazil, Costa Rica, Cyprus, France, Hungary, India, Republic of Korea, Lebanon, Mali, Nigeria, Portugal, United States, Yugoslavia and Zaïre. Indirect taxes minus subsidies are not shown as a distinct category of value added.

22/ Sweden. A cross-classification of commodity taxes and trade and transport margins is used to construct a commodity x commodity input-output table at basic values but is not published.

Table 4. Tables derived from comprehensive input-output tables

Country	Year to which the comprehensive input-output table relates	Derived matrix of direct input-output coefficients	Derived matrices of cumulative (inverse) input-output coefficients: Direct and indirect supply needed per unit of final demand: Domestic production and imports	Derived matrices of cumulative (inverse) input-output coefficients: Direct and indirect supply needed per unit of final demand: Domestic production only	Derived matrices of cumulative (inverse) input-output coefficients: Direct and indirect use of primary inputs: Per unit of gross output	Derived matrices of cumulative (inverse) input-output coefficients: Direct and indirect use of primary inputs: Per unit of each final demand category	Input-output tables after triangulation
		1	2	3	4	5	6
1. Algeria	1954						
2. Argentina	1960	x		x			
3. Australia	1958/59	x	x	x			
4. Austria	1961						
5. Belgium	1965						
6. Bolivia	1958						
7. Brazil	1959						
8. Bulgaria	1963						
9. Canada	1961	x	x	x	x	x	
10. Ceylon	1965	x	x				
11. Costa Rica	1957						
12. Cyprus	1957						
13. Czechoslovakia	1962	x		x		x	
14. Denmark	1953	x		x	x		
15. Egypt	1962	x		x	x	x	
16. Fiji	1967	x					
17. Finland	1965	x		x			
18. France	1962						
19. Germany, Federal Republic of	1966						
20. Greece	1960						
21. Guyana	1959						

Table 4. Tables derived from comprehensive input-output tables (continued)

Country	Year to which the comprehensive input-output table relates	Derived matrix of direct input-output coefficients	Derived matrices of cumulative (inverse) input-output coefficients: Direct and indirect supply needed per unit of final demand: Domestic production and imports	Derived matrices of cumulative (inverse) input-output coefficients: Direct and indirect supply needed per unit of final demand: Domestic production only	Derived matrices of cumulative (inverse) input-output coefficients: Direct and indirect use of primary inputs: Per unit of gross output	Derived matrices of cumulative (inverse) input-output coefficients: Direct and indirect use of primary inputs: Per unit of each final demand category	Input-output tables after triangulation
		1	2	3	4	5	6
22. Hungary	1968	x	x	x	x		
23. India	1964/65						
24. Iran	1965						
25. Iraq	1963						
26. Ireland	1964	x		x			
27. Israel	1968/69	x	x	x[1]	x	x	
28. Italy	1967	x	x	x	x		
29. Ivory Coast	1960	x	x				x
30. Jamaica	1958						
31. Japan	1965	x	x	x	x	x	
32. Jordan	1968						
33. Korea, Republic of	1968	x	x[2]	x[2]			
34. Lebanon	1964						
35. Malawi	1968						
36. Malaysia	1960	x		x			
37. Mali	1958	x	x				x
38. Malta	1968	x		x	x	x	
39. Mexico	1960						
40. Morocco	1968						
41. Netherlands	1964	x		x	x	x	
42. New Zealand	1959/60	x	x	x	x	x	
43. Nigeria	1959/60	x		x			
44. Norway	1964	x		x	x	x[3]	
45. Pakistan		x[4]		x[4]			

Table 4. Tables derived from comprehensive input-output tables (continued)

Country	Year to which the comprehensive input-output table relates	Derived matrix of direct input-output coefficients	Derived matrices of cumulative (inverse) input-output coefficients				Input-output tables after triangulation
			Direct and indirect supply needed per unit of final demand		Direct and indirect use of primary inputs		
			Domestic production and imports	Domestic production only	Per unit of gross output	Per unit of each final demand category	
		1	2	3	4	5	6
46. Peru	1959	x					
47. Philippines	1961	x	x	x			
48. Poland	1962	x	x	x			
49. Portugal	1959	x	x				x
50. Senegal	1959	x					
51. South Africa	1956/57	x		x			x
52. Spain	1962						
53. Sudan	1961/62						
54. Sweden	1964						
55. Togo	1958						
56. Trinidad and Tobago	1962	x					
57. Tunisia	1957						
58. Turkey	1963	x	x				
59. Union of Soviet Socialist Republics	1966	x	x				
60. United Kingdom	1963	x[5]		x[5]	x[5]	x[5]	
61. United States	1963	x	x				
62. Yugoslavia	1966	x	x	x			
63. Zaïre	1957						
64. Zambia	1967						

See the following pages for footnotes to table 4.

Footnotes to table 4

1/ Israel. The coefficients of direct and indirect domestic supply per unit of final demand are compiled from the difference between the coefficients for total supply and for imports. The coefficients for total supply and imports, but not for domestic supply, are published.

2/ Republic of Korea. Three sets of coefficients are compiled. They are based on input-output tables which include respectively all imports, competitive imports only and no imports in intermediate consumption.

3/ Norway. The direct and indirect needs for primary inputs are specified for each of a large number of categories of final demand.

4/ Pakistan. The original input-output table has 30 categories of industries and commodities; the coefficient matrix and its inverse has only 13 categories.

5/ United Kingdom. Two matrices of input-output coefficients are published - one for a commodity x commodity table and the other for an industry x industry table. The coefficients of direct and indirect supply indicated in columns 3 through 5 are based on the industry x industry input-output coefficient matrix.

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